CH 9: Hypothesis Testing Part 1

1. Introduction

   (A) The aim of testing statistical hypotheses is to determine whether a claim or conjecture about some feature of the population parameter (say, the mean μ, or the proportion p) is strongly supported by the information obtained from the sample data.

   (B) Some basic concepts in Hypothesis Testing

      (a) A set of hypotheses:
      H_1 or H_a : the claim or the research hypothesis that we wish to establish is called the alternative hypothesis.

      H_0 (Null hypothesis): Refers to a specified value of the population parameter.

      (b) There are three forms of Hypotheses in this chapter:

      Form 1: Two-tailed test
      H_0 : Parameter = reference value
      H_1 : Parameter $\neq$ reference value

      Form 2: Upper, one-tailed test
      H_0 : Parameter $\leq$ reference value
      H_1 : Parameter $>$ reference value

      Form 3: lower, one-tailed test
      H_0 : Parameter $\geq$ reference value
      H_1 : Parameter $<$ reference value

   (C) Type I and Type II error of the test

   (D) The probability of making a type I error= α: level of significance.

   (E) Test Statistic: A statistic whose value helps determine whether a null hypothesis should be rejected.

   (F) p-value: A probability that provides a measure of the evidence against the null hypothesis provided by the sample. If the p-value is less than α, we reject H_0; If the p-value is more than α, we fail to reject H_0.

2. Application: Hypothesis Testing

   Step 1: State H_0 vs. H_1.

   Step 2: Compute the test statistic

   Step 3: Compute the p-value based on the test statistic and making a decision:
   if the p-value is less than α, we reject H_0, otherwise, we fail to reject H_0.

(A) Case I: Z-test for the population mean μ (σ known)

$$Z_{cal} = \frac{\overline{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \tag{eq9.1}$$

Note: two-tailed test: $p - value = 2P(Z > |Z_{cal}|)$
upper, one-tail test: $p - value = P(Z > Z_{cal})$
lower, one-tail test: $p - value = P(Z < Z_{cal})$

EX 1. A manager wants to know if the amount of paint in 1-gallon cans is indeed 1-gallon. Given that the population standard deviation is 0.02 gallon. A random sample of 50 cans is selected and the sample mean is 0.995 gallon. Is there evidence that the mean amount is different from 1 gallon ($\alpha = 0.01$)?

(a) State H_0 and H_1

(b) Compute the test statistic

(c) Find the p-value and make a decision.

(B) Case II: t-test for the population mean μ (σ unknown)

$$t_{cal} = \frac{\overline{X} - \mu_0}{\frac{S}{\sqrt{n}}} \tag{eq9.2}$$

Note: use the t-table (with $n-1$ degrees of freedom) to obtain the range of the p-value and then make a decision.

EX 2. 100 candy bars are random selected with a mean of 1.466 and standard deviation of 0.132. For $\alpha = 0.05$, is there evidence that the average weight of the candy bars is less than 1.5 ounces?

(a) State H_0 and H_1

(b) Compute the test statistic

(c) Guessing the range of the p-value and make a decision.

CH 9: Hypothesis Testing Part 2

EX 2 (Cont) Is there evidence that the average weight of the candy bars is different from 1.5 ounces ($\alpha = 0.05$)?

(C) Case III: Z-test for the population proportion p

$$Z_{cal} = \frac{\overline{p} - p_0}{\sqrt{\frac{p_0\,(1-p_0)}{n}}} \qquad \text{(eq9.4)}$$

Note: two-tailed test: $p - value = 2P(Z > |Z_{cal}|)$
upper, one-tail test: $p - value = P(Z > Z_{cal})$
lower, one-tail test: $p - value = P(Z < Z_{cal})$

EX 3 It's claim that the usual percentage of overdraw is more than 10% on checking account(CA) at a bank. To test this claim, a random sample of 50 CA is examined and six out of 50 were found to be overdraw. What conclusion can you make at $\alpha = 0.05$?

(a) State H_0 and H_1

(b) Compute the test statistic

(c) Find the p-value and make a decision.

3. Making a decision based on the Critical Value

Step 1: State H_0 vs. H_1.

Step 2: Compute the test statistic and find the critical value.

Step 3: Make a decision based on the critical value.

EX 1 (cont) For the two-tail test, make a decision using the critical approach.

Step 1: State H_0 and H_1

Step 2: Compute the test statistic and find the critical value

Step 3: Make a decision.

EX 2 (cont) Use the critical value approach to test if the average weight of the candy bars is less than 1.5 ounces ($\alpha = 0.05$).

EX 3 (cont) Use the critical value approach to test the hypothesis.