Control Theory I: Applied Bifurcation Theory (Fall 2019)

Professor: Dr. Wenjing Zhang; Office: Math 214
Office Hours: Tuesday & Thursday: 10:00a.m.–12:00 p.m., or by appointment.

Class Meeting Time and Room: Tuesday & Thursday: 12:30 p.m.–1:50 p.m, Math 114

Course Website: http://www.math.ttu.edu/ wenjinzh/

Textbook: *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields* (by John Guckenheimer and Philip Holmes) and supplementary simulation materials.
Essential Prerequisites: single-variable calculus (curve sketching, Taylor series, and separable differential equations); multi-variable calculus (partial derivatives, Jocobian matrix, and divergence theorem); and linear algebra (eigenvalues and eigenvectors).

Tentative Course Agenda

- **Week 1 & 2 : Chapter 1–Introduction: Differential Equations and Dynamical Systems**

  Sec. 1.0 Existence and Uniqueness of Solutions
  Sec. 1.1 The Linear Systems $\dot{x} = Ax$
  Sec. 1.2 Flows and Invariant Sub-spaces
  Sec. 1.3 The Nonlinear System $\dot{x} = f(x)$

  - Nonlinear System & Linearization & Hartman's Theorem
  - **Lab 1:** Constructing Phase Plane Diagrams: direction fields, nullcines, and fixed points by Maple (or Mathematica) & XPPAUT
  - Applications of the planar system theory to modeling interacting species

  Home Work 1: Applications of the planar system theory to the following models: (1) two mode laser, (2) evolutionary dynamics of two interacting species, (3) model of a national economy, (4) hyper-cycle equation, (5) opinion dynamics: leftists, rightist, centrists

- **Week 3 & 4 : Chapter 1 Cont.**

  Sec. 1.4 Linear & Nonlinear Maps
  Sec. 1.5 Closed Orbits, Poincaré Maps, and Forced Oscillation

  - Poincaré Maps
  - Hamiltonian System in the Plane and with Two Degrees of Freedom
  - Non-autonomous Systems in the Plane: Forced Linear Oscillators & Duffing Equations
  - The Periodic Perturbed Pendulum
  - **Lab 2:** Projections of Poincaré Maps and Poincaré Section

  Home Work 2: For the planar systems, derive the analytical form of Poincaré Maps and plot it; for a two-degree-of-freedom Hamiltonian system, the plot 2-&3-dimensional Poincaré sections by Maple (&/or) Mathematica.

- **Week 5 : Chapter 1 Cont.**

  Sec. 1.6 Asymptotic Behavior
  Sec. 1.7 Equivalence Relations and Structural Stability

- **Week 6 & 7 : Chapter 1 Cont.**

  Sec. 1.8 Two-dimensional Flows

  - Conservative system, Lyapunov Functions, and Stability
  - **Lab 3:** The surface, contour and density plot of a Lyapunov function
  - Index Theory

Sec. 1.9 Peixoto's Theorem for Two-dimensional Flows
Home Work 3: Analytical and numerical analysis of (1) the bead's motion, (2) rock-paper-scissors problem, (3) the relation of rabbits vs foxes, and (4) the pendulum problem.

- **Week 8 : Chapter 2–An Introduction to Chaos:** Four Three-Dimensional Autonomous Systems and Chaos
  Sec. 2.1 (1) Van der Pol's Equation; (2) Duffing's Equation; (3) The Lorenz Equations; (4) The Rössler System
  Sec. 2.2 **Lab 4** Use the Maple (&/or) Mathematica package to plot phase portraits and time series, then investigate chaotic solutions.

- **Week 9 : Chapter 3: Local Bifurcations:**

  Sec. 3.1 Bifurcation Problems
  Sec. 3.2 Center Manifold: the computation for parametrized families of system via Maple
  Sec. 3.3 Normal Forms

- **Week 10 & 11 : Chapter 3 Cont.**

  Sec. 3.4 Co-dimension One Bifurcations of Equilibria Sec. 3.5 Co-dimension One Bifurcation of Maps and Periodic Orbits

  Lab 5: Computation of the analytical forms of (1) the center manifold for parametrized families of system, (2) static (saddle-node, transcritical, and pitchfork) bifurcation normal forms for high-dimensional systems, (3) Hopf bifurcation normal forms for high dimensional systems, via Maple.

  Home Work 5: Apply computational techniques from Lab 5 to high-dimensional physical systems.

- **Week 12 : Chapter 3 Cont.**

  Sec. 3.6 Introduction to Limit Cycles: (1) ruling out closed orbits; (2) Poincaré-Bendixon Theorem; (3) Liénard Systems; (4) Relaxation Oscillations; (5) Weakly Nonlinear Oscillators; (6)* Global Bifurcations of Cycles; (7)* Hysteresis in the Driven Pendulum and Josephson Junction; (8)* Coupled Oscillators and Quasi-periodicity.

  ∗ selected topics.

  Home Work 6: An application of Hopf bifurcation: analyze a chemical model for oscillations in the chlorine dioxide-iodine-malonic acid reaction.

- **Week 13: Chapter 4:** Averaging and Perturbation from a Geometric Viewpoint

  Sec. 4.1 Averaging and Poincaré Maps
  Sec. 4.2 Examples of Averaging
  Sec. 4.3 Averaging and Local Bifurcations
  Sec. 4.4 Averaging, Hamiltonian Systems, And Global Behavior: Cautionary Notes

- **Week 14 : Chapter 4 Cont.**

  Sec. 4.5 Melnikov's Method: Perturbations of Planar Homoclinic Orbits
  Sec. 4.6 Melnikov's Method: Perturbations of Hamiltonian Systems and Sub-harmonic Orbits

- **Week 15 : Chapter 4 Cont.**

  Sec. 4.7 Stability of Sub-harmonic Orbits
  Sec. 4.8 Two Degree of Freedom Hamiltonians and Area Preserving Maps of the Plane

  Home Work 7: Applying Melnikov-type integrals to determine the existence and maximum number of limit cycles in planar systems.

Expected Learning Outcomes: The goals of this course are to become familiar with the theory of nonlinear dynamics and chaos. 1) Students will review basic results in the relatively complete theory of two-dimensional differential equations. 2) Students will observe the chaotic behaviors induced by three-dimensional autonomous systems. 3) Students will learn about analytical and numerical computations for the local bifurcations. 4)

Students will lean the analytical methods of averaging and perturbation theory for the study of periodically forced nonlinear oscillators. 5) Students will learn the geometric approaches to dynamical systems and gain geometric intuitions. 6) Students will develop several Maple &/or Mathematica programs for algebraic manipulations and numerical simulations. 7) Students will also gain experience of the simulating and analyzing toolbox XPPAUT. 8) Students will learn the applications on classical mechanics, system biology, evolutionary biology, and socialphysics.

Methods of Assessment of Learning Outcomes: Continuous formative assessment of the progress of the course will occur via ongoing communication between the instructor and the students. To this end, all students are encouraged to ask questions during class and to seek the instructor's help outside of class when needed. Formal assessment occurs through homework and attendance. (See descriptions below.)

Assessment: Students will choose five out of the seven home work assignments. Each of the five submitted homeworks takes 20% of the final score.

Important Dates and Facts:

1. Classes Begin: Aug 26, 2019
2. Last Day of Classes: Dec. 4
3. Holidays : Sep 2 & Nov. 27Dec. 1
4. **Civility in the Classroom**: Texas Tech University is a community of faculty, students, and staff that enjoys an expectation of cooperation, professionalism, and civility during the conduct of all forms of university business, including the conduct of student–student and student–faculty interactions in and out of the classroom. Further, the classroom is a setting in which an exchange of ideas and creative thinking should be encouraged and where intellectual growth and development are fostered. Students who disrupt this classroom mission by rude, sarcastic, threatening, abusive or obscene language and/or behavior will be subject to appropriate sanctions according to university policy.
5. **Academic Integrity**: It is the aim of the faculty of Texas Tech University to foster a spirit of complete honesty and high standard of integrity. The attempt of students to present as their own any work not honestly performed is regarded by the faculty and administration as a most serious offense and renders the offenders liable to serious consequences, possibly suspension. Scholastic dishonesty includes, but it not limited to, cheating, plagiarism, collusion, falsifying academic records, misrepresenting facts, and any act designed to give unfair academic advantage to the student (such as, but not limited to, submission of essentially the same written assignment for two courses without the prior permission of the instructor) or the attempt to commit such an act.
6. **Observance of Religious Holy Day**: "Religious holy day" means a holy day observed by a religion whose places of worship are exempt from property taxation under Texas Tax Code 11.20. A student who intends to observe a religious holy day should make that intention known in writing to the instructor prior to the absence. A student who is absent from classes for the observance of a religious holy day shall be allowed to take an examination or complete an assignment scheduled for that day within a reasonable time after the absence. A student who is excused may not be penalized for the absence; however, the instructor may respond appropriately if the student fails to complete the assignment satisfactorily.
7. **Accommodation of Students with Disabilities**: Any student who, because of a disability, may require special arrangements in order to meet the course requirements should contact the instructor as soon as possible to make any necessary arrangements. Students should present appropriate verification from Student Disability Services during the instructors office hours. Please note: instructors are not allowed to provide classroom accommodations to a student until appropriate verification from Student Disability Services has been provided. For additional information, please contact Student Disability Services in West Hall or call 806-742-2405.