 # Strogatz Page 83, Prob. 3.4.14: Subcritical pitchfork

```
> restart:with(LinearAlgebra):with(plots):
> fr:=r*x+x^3-x^5;
```

$$fr := -x^5 + x^3 + r\,x \tag{1}$$

```
> fr2:=factor(fr);
```

$$fr2 := x\,(-x^4 + x^2 + r) \tag{2}$$

```
> x1:=0:
> fr3:=-y^2+y+r;
```

$$fr3 := -y^2 + r + y \tag{3}$$

```
> y23:= solve(fr3,y);
```

$$y23 := \frac{1}{2} + \frac{\sqrt{1+4\,r}}{2},\ \frac{1}{2} - \frac{\sqrt{1+4\,r}}{2} \tag{4}$$

```
> y2:= y23[1];
```

$$y2 := \frac{1}{2} + \frac{\sqrt{1+4\,r}}{2} \tag{5}$$

```
> y3:=y23[2];
```

$$y3 := \frac{1}{2} - \frac{\sqrt{1+4\,r}}{2} \tag{6}$$

```
> # y3 exists if 0< 1+4*r <1
> # Dxfr
> Dxfr:= diff(fr,x);
```

$$Dxfr := -5\,x^4 + 3\,x^2 + r \tag{7}$$

```
> Dxfr0:= subs(x=x1,Dxfr);
```

$$Dxfr0 := r \tag{8}$$

```
> Dxfr23:=subs(x=sqrt(y),Dxfr);
```

$$Dxfr23 := -5\,y^2 + r + 3\,y \tag{9}$$

```
> Dxfr2:= subs(y=y2,Dxfr23);
```

$$Dxfr2 := -5\left(\frac{1}{2} + \frac{\sqrt{1+4\,r}}{2}\right)^2 + r + \frac{3}{2} + \frac{3\sqrt{1+4\,r}}{2} \tag{10}$$

```
> Dxfr3:= subs(y=y3,Dxfr23);
```

$$Dxfr3 := -5\left(\frac{1}{2} - \frac{\sqrt{1+4\,r}}{2}\right)^2 + r + \frac{3}{2} - \frac{3\sqrt{1+4\,r}}{2} \tag{11}$$

```
> Dxfr2:=factor(Dxfr2)
```

$$Dxfr2 := -1 - \sqrt{1+4\,r} - 4\,r \tag{12}$$

```
> Dxfr3:=factor(Dxfr3)
```

$$Dxfr3 := -1 + \sqrt{1+4\,r} - 4\,r \tag{13}$$

```
> # if y2 exists, it is stable
> # if y3 exists, it is unstable
```

```
> implicitplot(fr,r=-1/3..1/3,x=-3..3,numpoints=10^5,thickness=6)
```