Asymptotic behavior of a low-dimensional model for magnetostriction for periodic input

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Abstract

Models for magnetostrictive actuators need to include rate-independent hysteresis phenomena, magneto-elastic coupling, and eddy current losses that vary nonlinearly with the frequency of the input. In this paper, we study a low dimensional model for magnetostrictive rod actuators that describes the physical phenomena which are most prominent in the frequency range 0-800Hz. We show that the solution of the system is asymptotically periodic for bounded, continuous and periodic voltage inputs and with general conditions on a Preisach operator modeling rate-independent hysteresis. The results of this paper are crucial for developing a parameter identification methodology for the model that is addressed in [9].

Key words: hysteresis, Preisach operator, eddy current losses, excess losses, asymptotically periodic solutions

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1 Introduction

Magnetostrictive actuators have two primary applications: precision positioning and vibration suppression. The objective in the second application is to cancel vibrations of a specific frequency in a host structure using magnetostrictive actuators. This is possible, as experiments show that for continuous, periodic voltage inputs in the frequency range 0 - 1 KHz and constant loading, the displacement of the tip of a magnetostrictive actuator is asymptotically a periodic function (see [1,2]). It is therefore necessary that any model for magnetostrictive actuators used for vibration suppression reproduce this behavior.

Most "low-dimensional models" [1] do not account for the metallic character of the magnetostrictive material in the sense that their electric conductivity is not incorporated in the model [4–6]. This means that important sources of power loss, namely eddy and excess losses, in the magnetostrictive material, the magnetic circuit and the electrical winding that produces the magnetic field, are not accounted for. These losses become significant when the frequency of the excitation increases beyond just 10 Hz [1,2], and hence their inclusion in the model is very important. A model which includes excess losses was studied in [8], following the work on soft ferromagnets in [7]. Existence and uniqueness of solutions were shown under general conditions on the Preisach operator that was used to model hysteresis. *Here, we prove analytically that the average strain in the actuator model proposed in [8] is asymptotically a periodic function for continuous, periodic voltage inputs and constant loads.* This analytically justifies the use of RMS quantities to identify parameters corresponding to excess losses and to obtain results on the frequency range

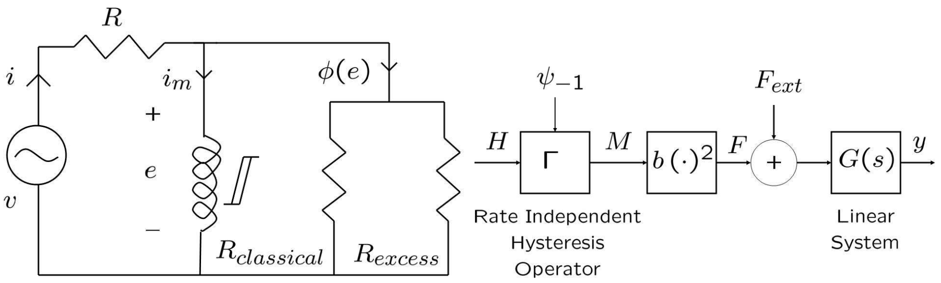
[1] Low dimensional models are so called because they are not PDE models

for which the model in [9] is valid.

2 The model

We summarize the low dimensional model for magnetostriction proposed in [1,8]. Figure 1(a) shows a three branch circuit. It models a magnetostrictive actuator connected to a voltage supply $v(\cdot)$, with a lead resistor R. The hysteretic inductor shown in one branch accounts only for rate-independent hysteresis losses. In the other branch, classical eddy current and excess losses are modeled using resistors $R_{classical}$ and R_{excess} respectively. Current i_m is proportional to the average magnetic field H in the axial direction for actuators and can be expressed as $i_m = k\,H$. Magnetic field H is related to the axial magnetization M via a rate-independent Preisach operator as $M(\cdot) = \Gamma[H(\cdot), \psi_{-1}]$, where $H(\cdot)$, $M(\cdot) \in C[0,T]$, and ψ_{-1} is the initial memory curve [3]. Voltage e across the inductor in Figure 1(a) depends on H and M via Lenz's law, as in Equation (2) below. Figure 1(b) models transduction from the magnetization to the strain in the axial direction for the actuator. The quantity $b\,M^2$ is a mechanical force F that, combined with external load F_{ext}, acts as input to the linear mechanical system yielding the strain of the magnetostrictive actuator.

As observed earlier, inclusion of eddy current and excess losses are crucial for producing models accurate over larger frequency ranges, such as 0-1kHz. Eddy current power loss per cycle is given by the classical expression: $P_{eddy} = C_0\, f$. Our experiments (the results of which are in [9]) show that the excess power loss per cycle could be approximated by the functional relation: $P_{excess}(f) = \sum_{i=1}^{N} C_i\, f^{\alpha_i}$, where $\frac{1}{2} \leq \alpha_i < 1$ for $i = 1, \ldots, N$. These losses are represented by nonlinear resistors $R_i(e) = C_i\,|e(t)|^{1-\alpha_i}$; $i = 1, \cdots, N$ in parallel. These re-



(a) A magnetostrictive actuator connected to a power supply.

(b) Model of the transduction of the magnetic field $H(\cdot)$ to the actuator displacement $y(\cdot)$.

Fig. 1. A low dimensional model for magnetostrictive actuators that accounts for eddy current and excess losses.

sistors are shown in Figure 1(a) by the equivalent resistor R_{excess}. The resistor $R_{classical}$ has a constant value. The current $\phi(e)$ in Figure 1(a) is then given by: $\phi(e) = \frac{R}{R_{classical}}\, e + R\,\mathrm{sign}(e(t)) \sum_{i=1}^{N} \frac{|e(t)|^{\alpha_i}}{C_i}$. This formula shows that $\phi(\cdot)$ is strictly monotone increasing with $\phi(0) = 0$.

The system of equations describing Figure 1(a) is given by:

$$\theta(e) + \beta H = v \tag{1}$$
$$\dot{B} = \gamma e \tag{2}$$
$$B(\cdot) = \mu_0\,(H + \Gamma[H(\cdot);\varphi_{-1}]) = \mathcal{P}[H(\cdot);\varphi_{-1}] \tag{3}$$
$$H(0) = H_0, \quad B(0) = B_0. \tag{4}$$

where H_0 and $B_0 = H_0 + \Gamma[H;\varphi_{-1}](0)$ are initial conditions. Parameters α, β, and γ are positive constants which depend on the given actuator. The continuous function $\theta(e)$ and operator $\mathcal{P}[H(\cdot);\psi_{-1}]$ are defined by: $\theta(e) = e + \alpha\phi(e)$ and $\mathcal{P}[H(\cdot);\varphi_{-1}] = \mu_0\,(\mathcal{I}+\Gamma)[H(\cdot);\varphi_{-1}]$ respectively, where $\mathcal{I}$ is the identity operator. The mechanical part of the model is given by [8]:

$$m\,\ddot{x} + c\,\dot{x} + k\,x = b\,M^2 + F_{ext}. \tag{5}$$

Brokate and Sprekels [3] show that a Preisach operator has an output map

$Q : \Psi_0 \to \mathbb{R}$ of the form (where Ψ_0 is the space of admissible memory curves):

$$Q(\varphi) = \int_0^\infty q(r, \varphi(r))dr + w_{00}, \; q(r,s) = 2\int_0^s \omega(r,\sigma)d\sigma,$$

with $w_{00} = \int_0^\infty \int_{-\infty}^0 \omega(r,s)dsdr + \int_0^\infty \int_0^\infty \omega(r,s)dsdr$. Here, $\omega \in L^1_{loc}(\mathbb{R}_+ \times \mathbb{R})$ is the Preisach density function. This representation will be used in the paper.

2.1 Global solution

The existence, stability and uniqueness of weak local solutions for (1)-(4) for $v \in L^2(0,T)$ for the case $P_{excess} = C_1 f^{\frac{1}{2}}$ can be found in [8] and the proof for the general case $P_{excess}(f) = \sum_{i=1}^N C_i f^{\alpha_i}$ can be found in [13]. We use the following general hypotheses:

($\mathcal{H}1$): $\phi(e)$ is continuous and strictly monotone increasing with $\phi(0) = 0$.

($\mathcal{H}2$): The Preisach operator $\Gamma[\cdot\,; \psi_{-1}]$ is continuous and piecewise monotone increasing on $C[0,T]$.

($\mathcal{H}3$): The Preisach density function has compact support.

($\mathcal{H}4$): There exists real numbers C^*, $C_* > 0$ such that $-C_* \leq Q(\varphi) \leq C^*$ for all $\varphi \in \Psi_0$. Furthermore, there exist $\varphi^*, \varphi_* \in \Psi_0$ such that $Q(\varphi^*) = C^*$ and $Q(\varphi_*) = -C_*$.

Note that, under these hypotheses, $\theta(\cdot)$ is continuous and strictly monotone increasing with $\theta(0) = 0$. Also $\mathcal{P}$ is continuous and piecewise strictly monotone increasing. Hypotheses $\mathcal{H}1$ - $\mathcal{H}3$ are needed for the local existence and uniqueness of weak solutions while $\mathcal{H}4$ is a sufficient condition needed for the global existence. We will next prove that $(e(t), H(t), B(t))$ is bounded for a bounded $v(t)$ and, hence, the above unique solution is global.

Lemma 2.1 *Let $\mathcal{H}1$ - $\mathcal{H}4$ hold. Suppose $v(t)$ is continuous and bounded for all*

$t \geq 0$. *Let the initial conditions for* $H(t)$ *and* $B(t)$ *be* H_0 *and* B_0*, respectively. Then there exists an interval* $[H_*, H^*]$ *such that if* $H(t) \in \mathbb{R} \setminus [H_*, H^*]$ *for* $t \in [0, T]$ *then* $|e(t)| \leq |g(t)|$ *on the same interval, where*

$$g(t) = v(t) - \beta \left[\gamma \int_0^t \lambda(\tau) g(\tau) d\tau + B_0 \right],$$

and $\lambda(\cdot)$ *is a continuous function on* $[0, T]$ *satisfying* $0 < \lambda(t) \leq 1$.

Proof: From $\mathcal{H}4$ there exists $\bar{H}^*$ such that if $H(t) > \bar{H}^*$ for some $t \in [0, T]$, then $B(t) = H(t) + C^*$ and $\theta(e)(t) = v(t) - \beta \left[\gamma \int_0^t e d\tau + B_0 - C^* \right]$ (from Equations (1)-(3)). From the boundedness of $v(t)$ and Equation (1), for any given $\delta > 0$, there exists an interval $I = (\hat{H}_\delta^*, \infty)$ such that $\theta(e) < -\delta$ if $H \in I$. Then from $\mathcal{H}1$ and the definition of $\theta(e)$, there exists $\epsilon_1 > 0$ such that $e < -\epsilon_1$. Let $(H_\delta^*, \infty) = (\hat{H}_\delta^*, \infty) \cap (\bar{H}^*, \infty)$. If $H(t) \in (H_\delta^*, \infty)$ then:

$$\begin{aligned} -\epsilon_1 > e(t) \geq \theta(e)(t) &= v(t) - \beta \left[\gamma \int_0^t e \, d\tau + B_0 - C^* \right] \\ &\geq v(t) - \beta \left[\gamma \int_0^t e \, d\tau + B_0 \right]. \end{aligned}$$

Similarly, there exists $\epsilon_2 > 0$ and an interval $J = (-\infty, H_{\delta *})$ such that $\theta(e) > \delta$ and $e > \epsilon_2$ when $H \in J$. Then:

$$\begin{aligned} \epsilon_2 < e(t) \leq \theta(e)(t) &= v(t) - \beta \left[\gamma \int_0^t e \, d\tau + B_0 + C_* \right] \\ &\leq v(t) - \beta \left[\gamma \int_0^t e \, d\tau + B_0 \right]. \end{aligned}$$

Hence, if $H(t) \in \mathbb{R} \setminus [H_{\delta *}, H_\delta^*]$, then $|e(t)| > \min\{\epsilon_1, \epsilon_2\}$ and $|e(t)| \leq |g(t)|$, where $g(t) := v(t) - \beta \left[\gamma \int_0^t e \, d\tau + B_0 \right]$. Picking $\lambda(t)$ such that $\epsilon < \lambda(t) \leq 1$, we get $e(t) = \lambda(t) g(t)$, where $\epsilon > 0$. The continuity of $\lambda(t)$ follows from Equation (1) and the continuity of $v(t)$ and $H(t)$. □

Theorem 1 *Let* $\mathcal{H}1 - \mathcal{H}4$ *hold. Suppose* $v(t)$ *is continuous and bounded for* *all $t > 0$. Then $e(t)$ is bounded for all $t > 0$.*

Proof: Let $f(t) = v(t) - \beta B_0$. Then $g(t) = f(t) - \nu \int_0^t \lambda(\tau) g(\tau)\, d\tau$. The solution of $g(t)$ is given by $g(t) = f(t) - \nu \int_0^t \lambda(\tau) f(\tau) e^{-\int_\tau^t \nu\lambda(s)\, ds}\, d\tau$. Hence:

$$\begin{aligned} |g(t)| &\leq \|f\|_{\infty,[0,t]} + \nu \|f\|_\infty \left| \int_0^t e^{-\int_\tau^t \nu\lambda(s)\, ds}\, d\tau \right| \\ &= |f\|_\infty \left[1 + \frac{1 - e^{-\nu\epsilon t}}{\epsilon} \right] < \infty \quad \text{for every } \epsilon > 0. \end{aligned} \tag{6}$$

Now, $\lim_{\epsilon \to 0} |g(t)| \leq \|f\|_\infty (1 + \nu\, t)$ which is still bounded on every bounded interval in $\mathbb{R}_+$. Hence $g(t)$ is bounded for every $t \in \mathbb{R}_+$. Then from Lemma 2.1, $e(t)$ is bounded even if $H(t) \in \mathbb{R} \setminus [H_*, H^*]$ on some interval $[0, T]$. But then $H(t)$ is bounded from Equation (1). From this we can conclude that $e(t)$ and $H(t)$ remain bounded for all t. □

3 Existence of a periodic solution

We show that, under periodic forcing, system (1)-(4) admits a periodic solution. We first need invertibility of θ.

Lemma 3.1 *Function θ^{-1} is well-defined and strictly monotone on $\mathbb{R}$. Furthermore, it is Lipschitz continuous with Lipschitz constant* 1.

The proof is a direct consequence of the Inverse Function Theorem and hypothesis $\mathcal{H}1$. Hilpert's Inequality ([3], page 134) plays a key role in the proof of the asymptotic periodicity of the solution; however, we need a slightly different right hand side. The Heaviside function H is defined by: $H(v) = 1$ if $v > 0$ and $H(v) = 0$ if $v \leq 0$. Given a continuous function $v(\cdot)$, define:

$$H_{RL}(v)(t) \triangleq \lim_{\delta \searrow 0} H(v(t + \delta)). \tag{7}$$

It can be checked easily that Hilpert's inequality remains valid when the function $H(v)(t)$ is replaced by $H_{RL}(v)(t)$. The "signum" function is defined by: sign $(v) = -1$ if $v < 0$, sign $(v) = +1$ if $v > 0$, and sign $(v) = 0$ if $v = 0$. The Heaviside function and the signum function are related by: sign $(v) = H(v) - H(-v), \quad v \in \mathbb{R}$. So, given a continuous function $v(\cdot)$, we can define the modified "signum" function:

$$S(v)(t) \stackrel{\triangle}{=} H_{RL}(v)(t) - H_{RL}(-v)(t). \tag{8}$$

Then we have the following lemma whose proof is almost obvious.

Lemma 3.2 *Let $u \in W^{1,1}(\mathbb{R}_+)$. Then for almost every $t \in \mathbb{R}_+$, we have:* $\frac{d}{dt}|u|(t) = \frac{du}{dt}(t)\ S(u)(t)$.

The following lemma is used in the proof of the asymptotic periodicity of the solution in the next section.

Lemma 3.3 *Let $u_1, u_2 \in W^{1,1}(\mathbb{R}_+)$ and $\varphi_{-1,1}$, $\varphi_{-1,2} \in \Psi_0$. Then*

$$\begin{aligned}
&\frac{d}{dt}|u_1(t) - u_2(t)| + \\
&\frac{d}{dt}\int_0^\infty |q(r, \mathcal{F}_r[u_1(\cdot);\varphi_{-1,1}]) - q(r, \mathcal{F}_r[u_2(\cdot);\varphi_{-1,2}])|(t)dr \\
&\leq \left(\frac{d}{dt}\left(\mathcal{P}[u_1(\cdot);\varphi_{-1,1}] - \mathcal{P}[u_2(\cdot);\varphi_{-1,2}]\right)(t)\right) S(u_1 - u_2)(t).
\end{aligned}$$

The proof is straightforward and follows from Hilpert's Inequality. Finally, we prove the asymptotic behavior of the solution for continuous and periodic voltage inputs.

Theorem 2 *Let $\mathcal{H}1 - \mathcal{H}4$ hold. Let v be bounded and continuous. Suppose there exists a T such that $v(t + T) = v(t)$. Then there exists unique functions $\widehat{e}$ and $\widehat{H}$ such that* *i.* $\widehat{e}(t+T)=\widehat{e}(t)$ *and* $\widehat{H}(t+T)=\widehat{H}(t)$ *for all* $t\geq 0$, *with* $\lim_{t\to\infty}|e(t)-\widehat{e}(t)|=0$ *and* $\lim_{t\to\infty}\left|H(t)-\widehat{H}(t)\right|=0$.

ii. For every initial memory curve $\psi_{-1}\in\Psi_0$, *there exists* $\widehat{\psi}_{-1}\in\Psi_0$ *depending only on* v *and* ψ_{-1}, *such that* $\lim\limits_{t\to\infty}\left|B(t)-\mathcal{P}[\widehat{H}(\cdot),\widehat{\psi}_{-1}](t)\right|=0$.

Proof: Define the time-shifted quantities: $v^n(t)\triangleq v(t+nT)$, $H^n(t)\triangleq H(t+nT)$, $B^n(t)\triangleq B(t+nT)$, and $e^n(t)\triangleq e(t+nT)$ for any $n\in\mathbb{N}$. Also define time-shifted memory curves: $\varphi^n(t)(r)\triangleq\varphi(t+nT)(r)$. Next, for any $i,j\in\mathbb{N}$ with $i\neq j$, define the non-negative definite function:

$$V^{ij}(t)=|H^i-H^j|(t)+\int_0^\infty\left|q(r,\mathcal{F}_r[H^i;\varphi^i(0)])(t)-q(r,\mathcal{F}_r[H^j;\varphi^j(0)](t)\right|dr.$$

From Lemma 3.3 and Equations (1)-(3),

$$\dot{V}^{ij}(t)\leq(\frac{d}{dt}(\mathcal{P}[H^i;\varphi^i(0)]-\mathcal{P}[H^j;\varphi^j(0)])(t))S(H^i-H^j)(t)$$
$$=\gamma\,(\theta^{-1}[v^i-\beta H^i](t)-\theta^{-1}[v^j-\beta H^j](t))\,S(H^i-H^j)(t).$$

Note that if $H^i(t)<H^j(t)$ then $v(t)-\beta H^i(t)>v(t)-\beta H^j(t)$. This implies $\theta^{-1}[v^i-\beta H^i]-\theta^{-1}[v^j-\beta H^j]>0$. Similarly, if $H^i(t)>H^j(t)$, then $\theta^{-1}[v^i-\beta H^i](t)-\theta^{-1}[v^j-\beta H^j](t)<0$. Thus, if $H^i(t)\neq H^j(t)$ for any $t\in\mathbb{R}_+$ then $\dot{V}^{ij}(t)<0$. As $V^{ij}(t)$ is a monotone decreasing function that is bounded below, $\dot{V}^{ij}(t)\to 0$ as $t\to\infty$. Hence its upper bound must also satisfy: $\lim_{t\to\infty}\theta^{-1}[v^i-\beta H^i](t)-\theta^{-1}[v^j-\beta H^j](t)=0$. Then, by $\mathcal{H}1$, $\lim_{t\to\infty}[v^i-\beta H^i-v^j+\beta H^j](t)=0$, i.e. $\lim_{t\to\infty}[H^i-H^j](t)=0$, uniformly in t. For any $t\in\mathbb{R}_+$ define: $\widehat{H}(t)\triangleq\lim\limits_{n\to\infty}H^n(t)$. It is clear that the function $\widehat{H}$ is well defined. This function has the property $\widehat{H}(t+T)=\widehat{H}(t)$ by our proof above. If we define: $\widehat{e}\triangleq\theta^{-1}(v-\beta\,\widehat{H})$, then this function satisfies:

$\widehat{e}(t) = \lim_{n\to\infty} e^n(t)$. The uniqueness of $\widehat{H}$ and $\widehat{e}$ follows from the uniqueness of the solution.

To prove the second claim we will show that the sequence of memory curves $\{\varphi^n(0)\}$ converges to a single curve $\hat{\psi}_{-1} \in \Psi_0$. As this sequence forms a bounded, equicontinuous set, the uniform convergence of a subsequence follows from the Arzela-Ascoli theorem. We use Lemma 2.4.8 in [3] to show that the entire sequence is convergent. At time $t = n\,T$, the memory curve is given by $\varphi^n(0)$. If the input $H(\tau)$ from $[(n-1)T, nT]$ is re-applied to the operator $\Gamma[\,\cdot\,;\varphi^n(0)]$ during the interval $[nT,(n+1)T]$, then the resulting memory curve at $(n{+}1)T$ is again $\varphi^n(0)$. Repeating the above process on every interval $[(n+p-1)T,(n+p)T]$, $1 \le p \le k$ with $k \ge 1$, the corresponding memory curve at $(n+k)T$ is $\varphi^n(0)$. Denote the applied input by $H^{n-1}_{p,[0,T]}$. We can compare the resulting memory curve with the one that is obtained by applying the function $H(t)$ on $[nT,(n+k)T]$. By Lemma 2.4.8, of [3], we have:

$$\begin{aligned}
&\|\varphi^{n+k}(0) - \varphi^n(0)\|_\infty \le \\
&\quad \max\{\|H_{[nT,(n+k)T]} - H^{n-1}_{p,[0,T]}\|_\infty,\ \|\varphi^n(0) - \varphi^n(0)\|_\infty\} \\
&\quad = \max_{1\le p\le k} \|H^{n+p}_{[0,T]} - H^{n-1}_{[0,T]}\|_\infty.
\end{aligned}$$

By the uniform convergence of H^n on the interval $[0,T]$ to $\widehat{H}$, we see that, given any $\epsilon > 0$, we can pick an N such that for all n, $m > N$, we have: $\|\varphi^m(0) - \varphi^n(0)\|_\infty < \epsilon$. Hence, the sequence $\{\varphi^n\}$ is Cauchy and it converges to a function $\hat{\psi}_{-1} \in \Psi_0$.

Define: $\widehat{B}(t) = \Gamma[\widehat{H},\widehat{\varphi}](t);\ t \in [0,T]$. We have the following inequality from [3] (Prop. 2.4.9, page 58) where the norms are computed on $[0,T]$: $\|B^n - \widehat{B}\|_\infty \le$

$\max\{\|H^n - \widehat{H}\|_\infty, \|\varphi^n(0) - \widehat{\psi}\|_\infty\}$, which proves that $B^n(\cdot)$ converges to a function $\widehat{B}(\cdot)$ on $[0,T]$ which has the initial memory curve $\widehat{\psi}_{-1}$. Finally define $\widehat{x}$ as the solution to (5) with input $\widehat{M}$ and constant F_{ext}. Then the final claim follows from the stability property of the ODE (5) due to the assumptions on $m\,c$ and k. $\Box$

4 Conclusion

In this paper, we study a low dimensional model for magnetostrictive rod actuators that describes the physical phenomena which are most prominent in the frequency range 0-800Hz. We show that the solution of the system is asymptotically periodic for bounded, continuous and periodic voltage inputs. The results of this paper are crucial for developing a parameter identification methodology for the model that is addressed elsewhere.

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