Control Allocation and Compensation for Over-Actuated Systems with Non-Linear Effectors

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Abstract

Control allocation techniques for inversion-based control laws have evolved that ensure that commands to the control effectors do not violate rate or position limits of the individual effectors. These allocation techniques typically assume that all effector dynamics are fast relative to the system being controlled and are therefore neglected. Unfortunately there are practical cases where this assumption breaks down and it becomes desirable to compensate for lags between the commands to the effectors and the effector response. In this paper, a technique for compensating for individual effector dynamics while respecting actuator constraints is proposed.

1 Introduction

Previously, researchers have considered controller design issues for plants with magnitude and rate-limited inputs. In this paper, we consider controller design for the actuators themselves, and thus our "system" is magnitude and rate-limited at the output.

Recent advances in constrained control allocation techniques [1, 2] have enabled dynamic inversion control laws [3] to control over-actuated systems while ensuring that control effector rate and position limits are not violated. Much of this work has been aimed at the control of aerospace vehicles. This technique invariably assumes that the actuator dynamics are sufficiently fast relative to the plant dynamics such that the commanded effector positions are attained instantaneously. Dynamic compensation for the linear portion of these effector dynamics must be done carefully. Simply inserting the compensation into the pseudo-command ensures that rate and position limits will not be violated; however, each effector will receive the same amount of compensation. Since each effector may have a different bandwidth, the compensation must be applied individually. Simple filtering of the control allocation command may cause actuator rate and position limits to be violated. Closed loop dynamic compensation for slow linear dynamics of a rate and position limited effector is explored in this paper (Figure 1).

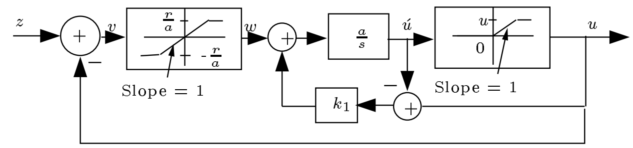


Figure 2: Schematic of control effector model

2 Constrained Control Allocation

It is assumed that a constrained control allocation algorithm [1, 2] is being used that ensures that a pseudo-command vector $\mathbf{d} = \mathbf{g}(\mathbf{u_c})$ is delivered by some combination of control effector deflections $\mathbf{u_C}$ subject to $\underline{\mathbf{u}} \leq \mathbf{u_c} \leq \overline{\mathbf{u}}$ where $\underline{\mathbf{u}}$ and $\overline{\mathbf{u}}$ are the most restrictive of the effector rate and position limits at any given instant. The actual control effector position for each actuator u is physically limited and always satisfies

$$u_{min} \leq u \leq u_{max}; \tag{1-a}$$
$$-r \leq \dot{u} \leq r. \tag{1-b}$$

where u is the magnitude limit and r is the rate limit for the desired actuator position signal u_c. Without loss of generality, we consider the case when $u_{min} = 0$ as in Figure 2.

Figure 2 shows the actuator schematic [4]. One can verify that the output signal in the block diagram indeed satisfies the constraints. The control allocation algorithm [3] takes care of these constraints 1-a - 1-b) and only those signals that satisfy the above constraints are passed as desired signals $u_c(\cdot)$ to the actuator. However for the sake of analysis, it is important to note that the input signal space to the actuator itself is much larger than the set of signals that satisfy the constraints 1-a - 1-b). We come to this point later.

2.1 Compensator design

Assuming that the effector position is a measurable quantity, u may be fed-back and used to form an error signal $e = u - u_c$ that may be used for dynamic compensation. We show in this paper that a simple compensator $K(s) = k\frac{s+a_1}{s}$ shown in Figure 1 satisfies

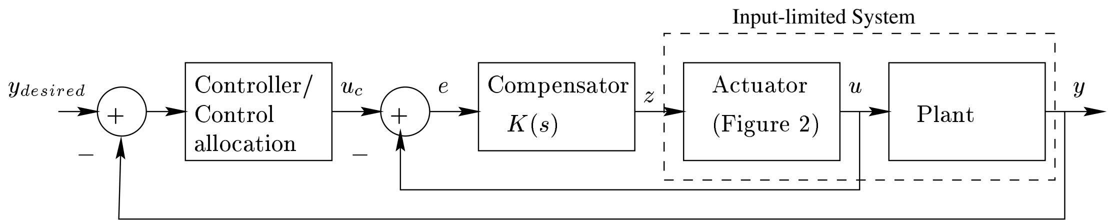


Figure 1: Schematic of the actuator and compensator

the requirements. We show that the error between the desired and actual effector positions can be made arbitrarily small by choosing k large enough. To be precise, we claim the following:

1. We choose $a_1 = a$ to simplify the analysis. If the initial states are zero and $u_c(\cdot)$ is the worst-case commanded input with $u_c(0) = 0$, the error $|e(t)| = |u_c(t) - u(t)| \le \frac{r}{k\,a}$ for all t. Thus to keep the error $|e(t)| \le \epsilon$, we simply choose $k \ge \frac{r}{a\,\epsilon}$.
2. If the initial states are non-zero and $u_c(\cdot) = 0$, then $u \to 0$ asymptotically.
3. If $u_c(0) \ne 0$, then we have $|e(t)| \le \frac{r}{k\,a}$ asymptotically.

Due to space constraints, we will indicate the main points of the proofs without giving all the details. As indicated in the list above, there are several cases to study. First of all we point out that given a commanded deflection $u_c(\cdot)$ that satisfies the magnitude and rate constraints, there are two important cases to consider. The first is when $u_c(0) = 0$ (item 1 above), and the second is when $u_c \ne 0$ (item 3). Other cases pertain to non-zero initial states (item 2) and choice of the compensator zero, which is not done here due to space constraints.

Proofs :

It is assumed that $u(t) = \acute{u}(t)$ for all t - this models the physical phenomenon. However, $w(t)$ may not be equal to $v(t)$ for all t.

1. As $u_c(0) = 0$ and the initial states are zero, we have $e(0) = u_c(0) - u(0) = 0$. Thus at $t = 0$, $v(0) = 0$. Thus by continuity, there exists an open interval of time $I = (0, \acute{t})$ such that $w(\tau) = v(\tau)$ for all $\tau \in I$. This means that for $t \in I$ we can ignore the non-linearities and consider the linear transfer functions alone. After some calculations we see that for $t \in I$: $\dot{e} = \dot{u_c} - kae$ and so,

$$e(t) = \int_0^t \exp\left(-ka(t-\tau)\right)\dot{u_c}(\tau)d\tau \qquad (2)$$

We note from Equation 2 that the closed-loop compensator plus effector system is well-defined for inputs $u_c(\cdot)$ that belong in the set $\mathcal{F} = \{f | f \in L^1_E[0,\infty) \& \dot{f} \in L^1_E[0,\infty)\}$. Now the control allocation algorithm only produces signals u_c that satisfy the constraints (1-a - (1-b). However, we show below that the closed-loop system is capable of tracking signals that satisfy the constraints at almost all points (except a set of measure zero).

From (2), it is clear that $|e(t)|$ is maximized if $|\dot{u_c}(t)| = r$ for almost all $t \in I$. In other words, the worst case error is obtained when $u_c(\cdot)$ is defined as follows. First pick T such that $r\,\frac{T}{2} \le u_{max}$. Then define :

$$\begin{aligned} u_c(t) &= rt, \;\; 0 \le t \le \frac{T}{2}; && \text{(3-a)} \\ &= u_c(\frac{T}{2}) - r(t - \frac{T}{2}), \;\; \frac{T}{2} \le t \le T && \text{(3-b)} \\ &= u_c(t - nT), \text{ for } nT \le t \le (n+1)T && \text{(3-c)} \end{aligned}$$

where $n \in \mathbb{N}$. As $e(0) = 0$, for $t \in J = I \cap [0, \frac{T}{2}]$,

$$e(t) = \frac{r}{k\,a}\left(1 - \exp\left(-k\,a\,t\right)\right) \qquad (4)$$

Next we compute the signals $u(\cdot)$, and $v(\cdot)$ in J (Figures 1 and 2).

$$\begin{aligned} u(t) &= rt - \frac{r}{ka}\left(1 - \exp\left(-kat\right)\right) && \text{(5-a)} \\ v(t) &= \frac{r}{a}\left(1 - \exp\left(-k\,a\,t\right)\right) && \text{(5-b)} \end{aligned}$$

Observing Equation (5-b), we can say that $|v(t)| < \frac{r}{a}$ for all $t \in [0, \frac{T}{2}]$ if $u(t)$ never achieves its upper bound u_{max}. we can conclude from the definition of T, Equations (3-a) and (5-a), that for all $t \in [0, \frac{T}{2}]$, $u(\cdot) < u_{max}$.

Thus $\forall\ t \in [0, \frac{T}{2}]$, both $v(\cdot)$ and $\acute{u}(\cdot)$ take values in $(\frac{r}{a}, \frac{r}{a})$ and $[0, u_{max})$ respectively. Observe that $v(t) = k\,v(t)$.

Next, we analyze the signals in $[\frac{T}{2}, T]$. By direct computations, we see that $\forall\, t \in [\frac{T}{2}, T]$; $|v(t)| < \frac{r}{a}$. We also observe that $v(t) = k\,e(t)$ for $t \in [\frac{T}{2}, T]$.

By similarly analyzing the signals in the time intervals $[n\,T, (n+1)\,T]$ for $n \in \mathbb{N}$, we arrive at the same conclusion that the signals v and $\acute{u}$ do not violate their limits (except for $\acute{u}(0) = 0$!). *This conclusion does not depend on the parameter* k (as long as $k > 0$!).

The magnitude of the error is determined by k. If we desire the error to be bounded by ϵ then we need to pick k such that

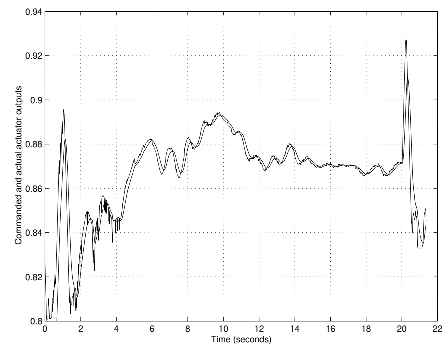
$$k \geq \frac{r}{a\,\epsilon} \tag{6}$$

2. If the initial states are non-zero and $u_c = 0$, then we have two cases. First is the case when the rate and magnitude limits are not reached. In this case, as we have a stable linear system, the states decay to zero asymptotically. This implies that the output $u \to 0$ asymptotically. In the second case, when the rate and magnitude limits are reached, then we can show with some calculations that the states decay so that the signals v and $\acute{u}$ eventually lie in $(-\frac{r}{a}, \frac{r}{a})$ and $(0, u_{max})$ respectively. Then we have the same situation as the first case. Thus again the output $u \to 0$ asymptotically.

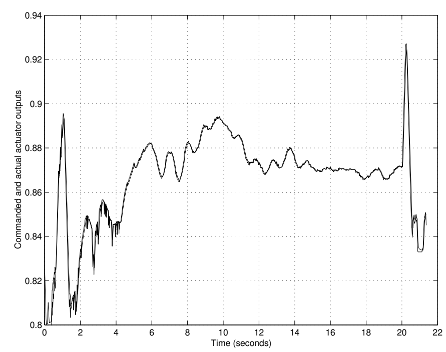
3. Here we consider the case when $u_c(0) \neq 0$. For simplicity, we examine the case when $u_c(\cdot) = c$ is a constant function of time. Here subtleties concerning the anti-windup circuit in Figure 2 enter, and so we consider the generic case of $c < u_{max}$ only. Consider the case when v is greater than $\frac{r}{a}$ during an interval of time (t_1, t_2). The case when v becomes less than $-\frac{r}{a}$ is handled similarly. Note that it is possible (if c is small enough) that the v and $\acute{u}$ always lie in $(-\frac{r}{a}, \frac{r}{a})$ and $(0, u_{max})$ respectively. In (t_1, t_2) we can write down the system equations and then after some minor calculations we observe that there must exist a time $t_* \in (t_1, t_2)$ such that $v(\cdot)$ decreases at least linearly in (t_*, t_2). Thus t_2 cannot be ∞ and thus $v(\cdot) \leq \frac{r}{a}$ for $t \geq t_2$. Then we have the same situation as in item 1. $\square$

3 Simulation results

In this section, we present simulation results that support the conclusions made earlier. For one of the control effectors, the parameters where : $r = 2.1$; $a = 7$; $u_{max} = 1.14$. Figure 3(a) shows the desired versus actual control effector positions without any compensator. Figure 3(b) shows the same signals with the compensator discussed above with $k = 7.5$. It was further verified that the relationship between k and ϵ holds.



(a) No compensation



(b) With compensation

Figure 3: Desired vs actual control effector positions.

4 Conclusion

In this paper, we study a compensator design problem for a system with rate and magnitude limits on the output. The desired signal also satisfies the same constraints. The problem is to design a compensator that accounts for the dynamics of the system. We have studied a simple compensator design and derived a relationship between the gain of the compensator and the desired error between the input and output signals. Simulations show the effectiveness of the compensator.

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