THETA FUNCTIONS AND KNOTS

Răzvan Gelca

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based on joint work with Alejandro Uribe and Alastair Hamilton

B. Riemann: **Theorie der Abel'schen Funktionen**

Riemann's work on elliptic integrals (based on insights of Abel and Jacobi):

Study integrals

$$u(x) = \int R(x, y)dt$$

where $y(x)$ is defined by a polynomial equation $P(x, y) = 0$ (elliptic for P cubic or cuartic).

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Study line integrals

$$u(x) = \int_a^x R(z(t), w(t))dt$$

where $w(z)$ is defined by a polynomial equation $F(z, w) = 0$.

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Study line integrals

$$u(x) = \int_a^x R(z(t), w(t))dt$$

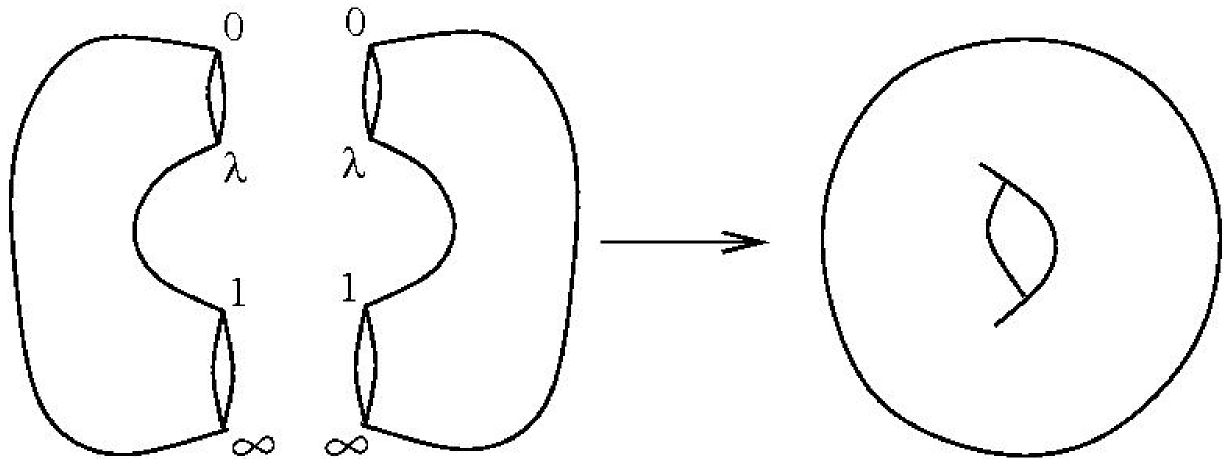
where $w(z)$ is defined by a polynomial equation $F(z, w) = 0$.

Because w lives naturally on a Riemann surface.

Example: For the Weierstrass curve

$$w^2 = z(z-1)(z-\lambda)$$

the Riemann surface is a torus:



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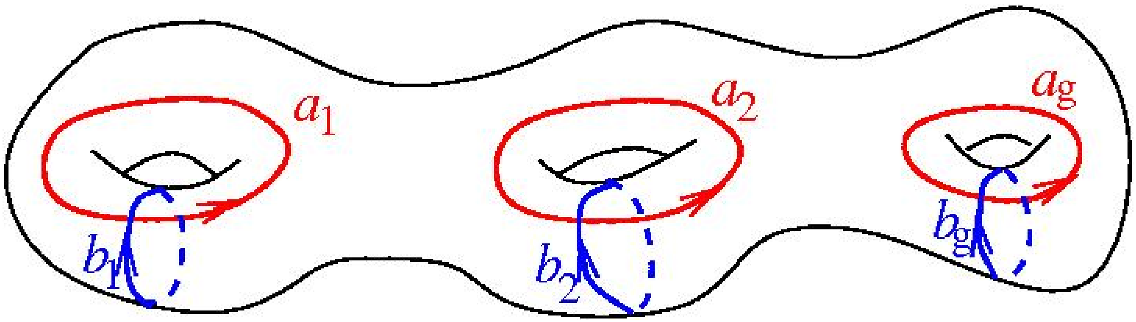
The building blocks of elliptic functions are theta functions.

• *Riemann surface*



$\zeta_1, \zeta_2, \ldots, \zeta_g$ *basis for* $\mathrm{Hol}^1(\Sigma_g)$, *such that* $\int_{a_j} \zeta_k = \delta_{jk}$, *and* Π *is the matrix with entries* $\int_{b_j} \zeta_k$.

• *Jacobian variety*

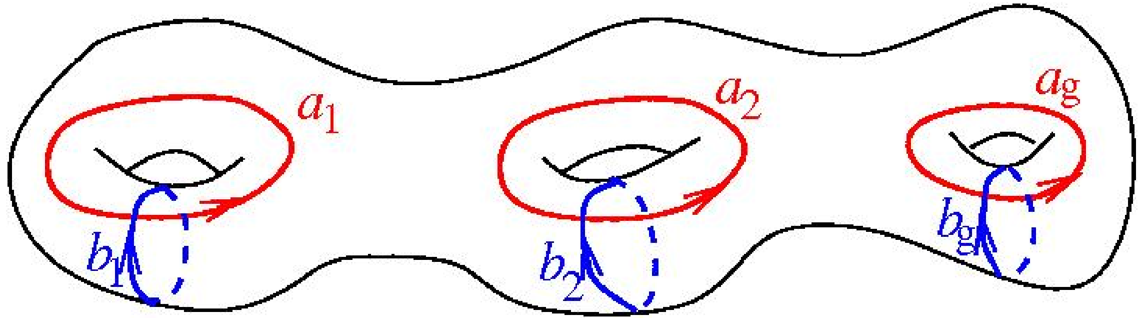
$$\mathcal{J}(\Sigma_g) = \mathbb{C}^g / \textit{span of columns of } (I_g, \Pi).$$

• *Riemann's theta series:*

$$\theta(z) = \sum_{n \in \mathbb{Z}^g} e^{2\pi i [\frac{1}{2} n \cdot \Pi n + n \cdot z]}.$$

(*where* $x \cdot y = x^T y = x_1 y_1 + x_2 y_2 + \cdots + x_n y_n$.)

- *Riemann surface*



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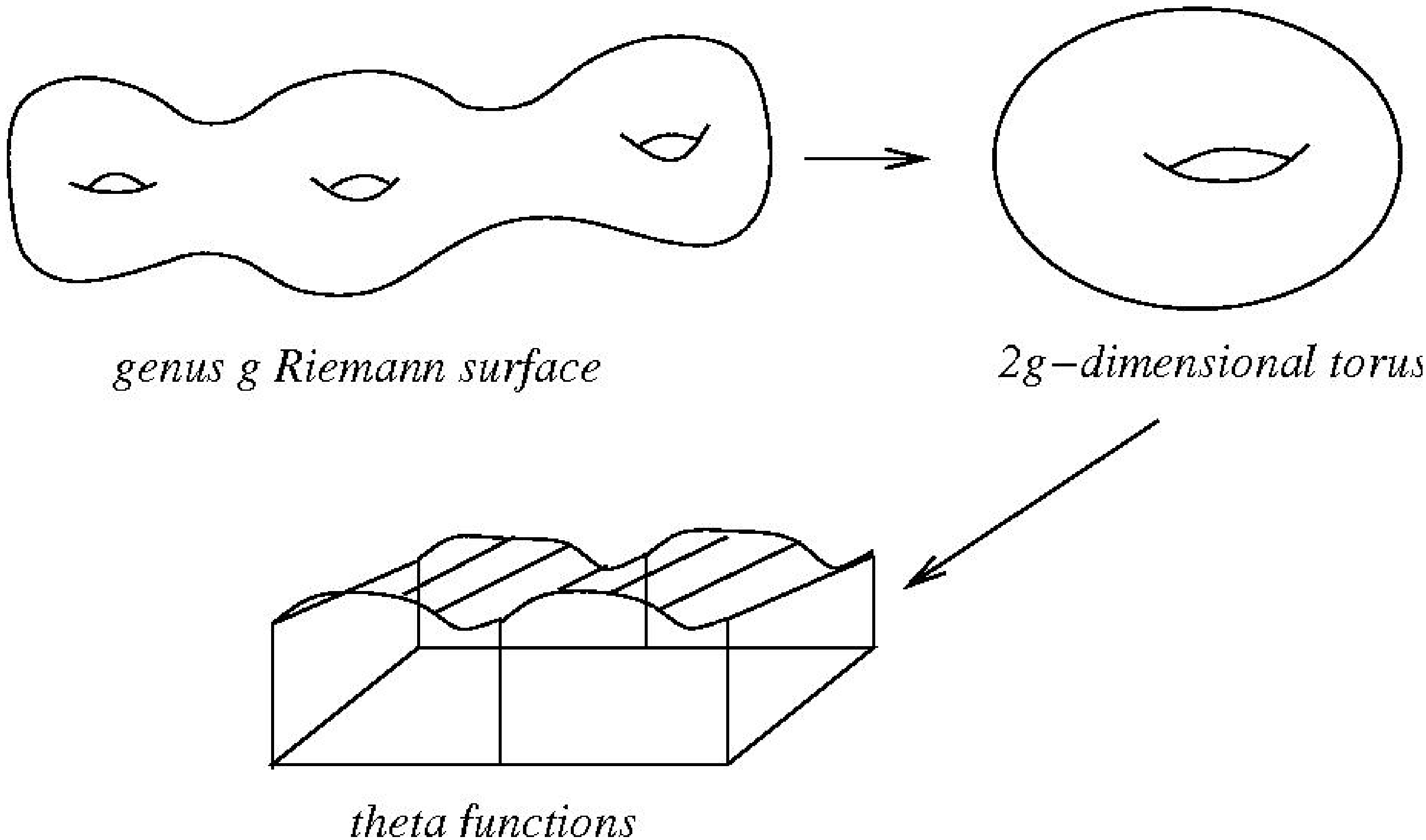
Classical theta functions



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- *one that arises from the symmetries of the Riemann surface*

Carl G.J. Jacobi

There are two group actions on the space of theta functions

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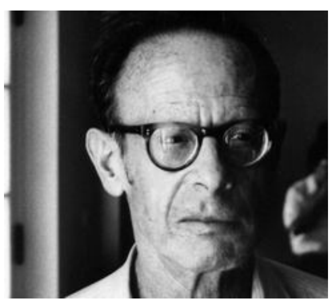
- *and one that arises from the group law on the Jacobian torus*



André Weil

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- *theta functions are states (aka wave functions, they are vectors in the Hilbert space of the quantization)*

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- *the second group action arises from the quantization of exponentials on the Jacobian variety (the quantized exponentials are linear operators on the Hilbert space)*

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This is similar to the quantization of several free particles (the Schrödinger representation and the metaplectic representation). It arises from quantizing g one-dimensional particles with periodic positions and momenta.

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- *theta functions are states*
- *the second group action arises from the quantization of exponentials on the Jacobian variety*
- *the first group action arises from the quantization of changes of coordinates.*

Planck's constant is $\frac{1}{N}$, N *even integer. The classical phase space is the Jacobian variety associated to a genus* g *surface (which is a* $2g$*-dimensional torus).*

States: linear combinations of

$$\theta_\mu(z) = \sum_{n \in \mathbb{Z}^g} e^{2\pi i N[\frac{1}{2}(\frac{\mu}{N}+n)\Pi(\frac{\mu}{N}+n)+(\frac{\mu}{N}+n)z]}, \quad \mu \in \mathbb{Z}_N^g.$$

Operators: quantized exponentials

$$\mathit{Op}\left(e^{2\pi i(px+qy)}\right)\theta_\mu = e^{\frac{\pi i}{N}pq - \frac{2\pi i}{N}\mu q}\theta_{\mu+p}.$$

States - obtained via geometric quantization:

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$$\mathit{Op}\left(e^{2\pi i(px+qy)+\frac{\pi i}{N}k}\right)\theta_\mu = e^{\frac{\pi i}{N}pq-\frac{2\pi i}{N}\mu q+\frac{\pi i}{N}k}\theta_{\mu+p}.$$

This is the action of a finite Heisenberg group.

The finite Heisenberg group, which we denote by $\mathbf{H}(\mathbb{Z}_N^g)$, *is a quotient of*

$$\{(p, q, k) \mid p, q \in \mathbb{Z}^g, k \in \mathbb{Z}\}$$
$$(p, q, k)(p', q', k') = (p + p', q + q', k + k' + pq' - qp')$$

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<u>Theorem</u> ("Stone-von Neumann") The representation of $\mathbf{H}(\mathbb{Z}_N^g)$ *on theta functions is the unique unitary irreducible representation of this group in which* $(0,0,k)$ *acts as multiplication by* $e^{\frac{\pi i}{N}k}$.

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Corollary Each homeomorphism of the Riemann surface induces a unitary map on theta functions. This gives rise to the action of the modular group on theta functions.

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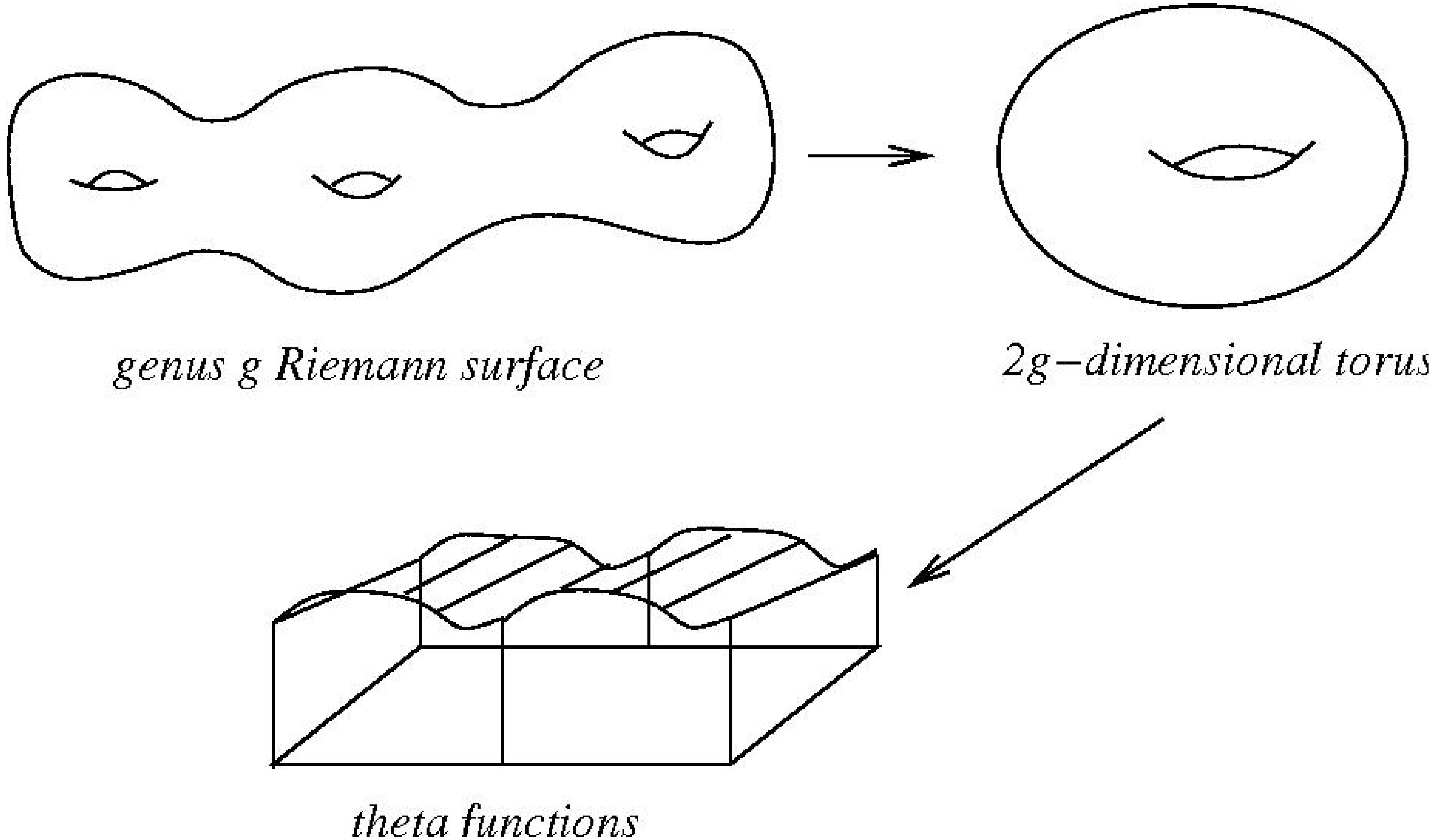
Observation (G.-Uribe): *This theorem implies that all the information about the action of the quantized exponentials and the action of the mapping class group of the surface is contained in* $\mathbf{H}(\mathbb{Z}_N^g)$.


genus g Riemann surface
2g−dimensional torus
theta functions

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$$e^{2\pi i(px+qy)} \mapsto (p,q) \in \mathbb{Z}^{2g} = H_1(\Sigma_g, \mathbb{Z}),$$

where the equality $\mathbb{Z}^{2g} = H_1(\Sigma_g, \mathbb{Z})$ *is realized using the cannonical basis on which periods are computed.*

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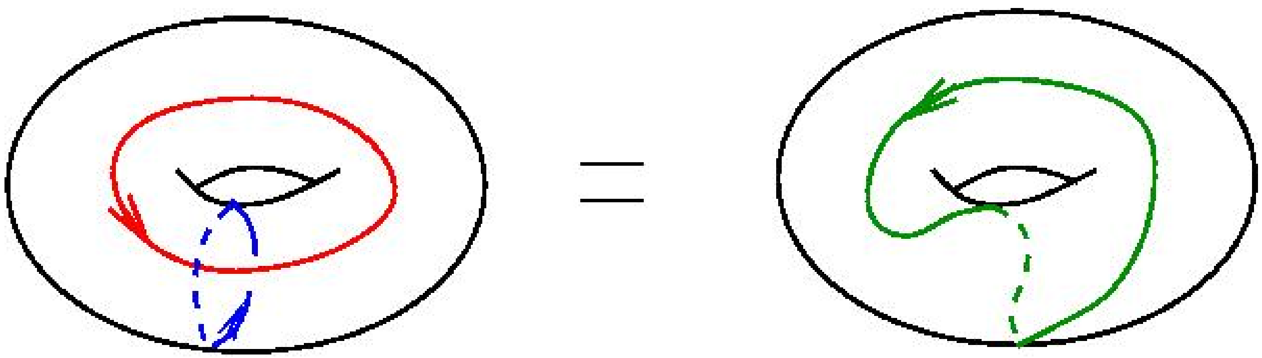

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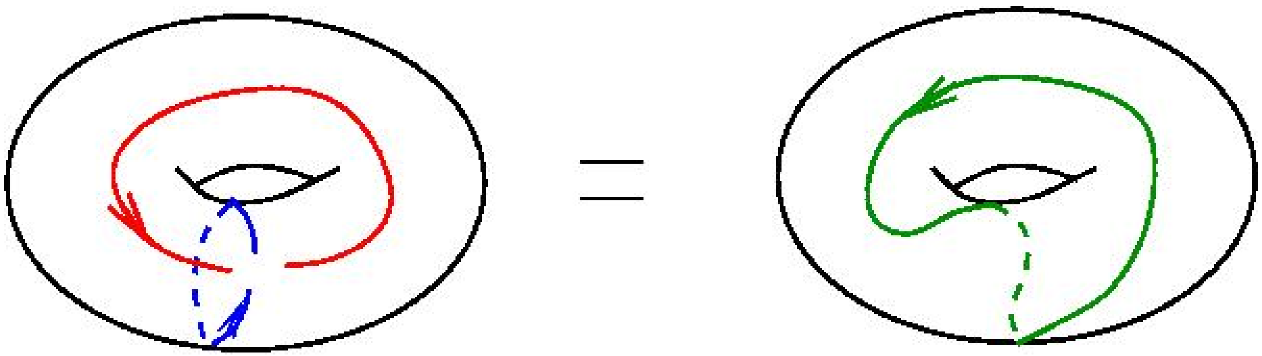

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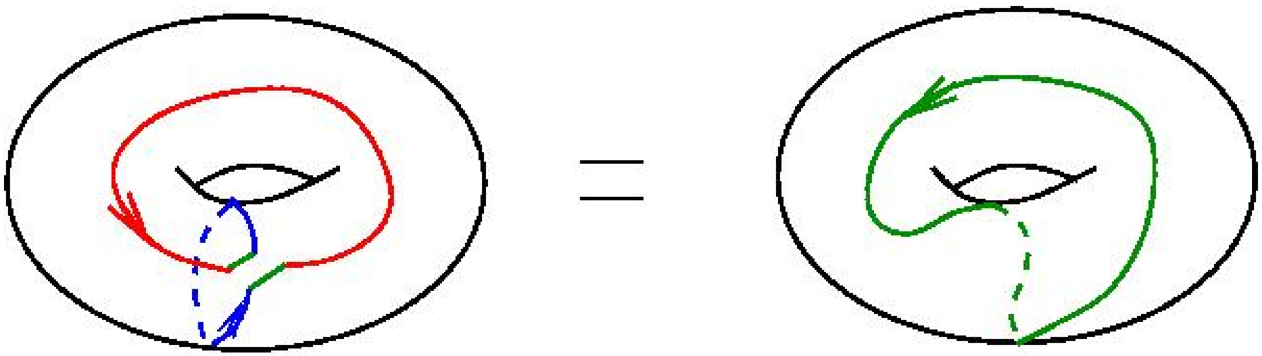
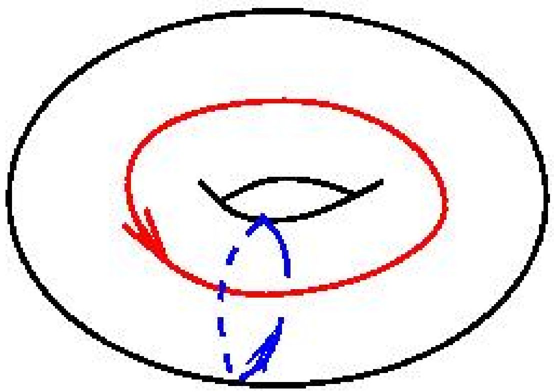
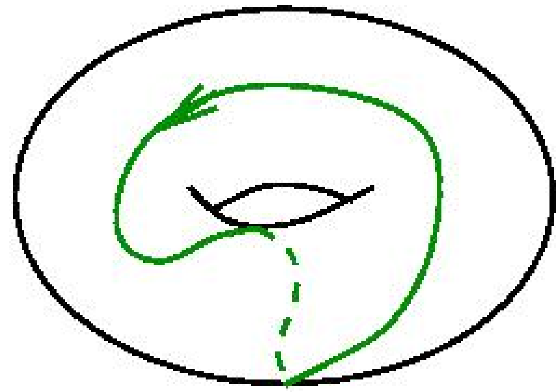


Multiplication in the Heisenberg group



$= e^{\frac{\pi i}{N}}$



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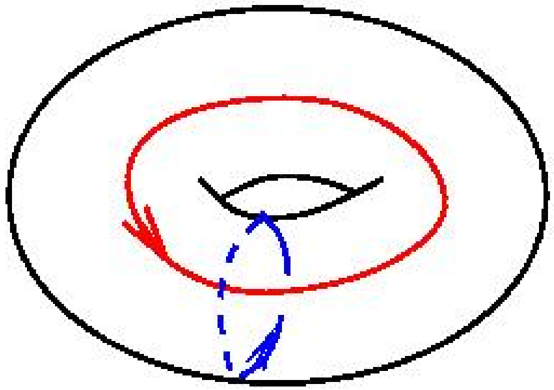

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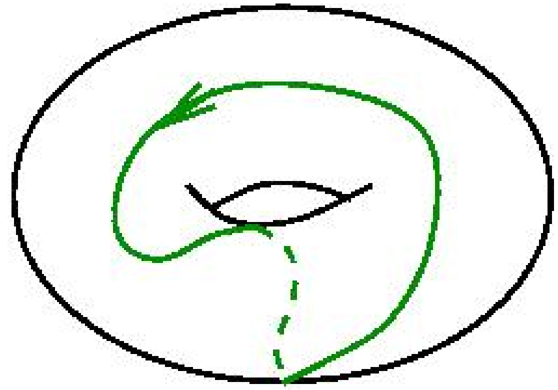

$$\mathit{Op}\left(e^{2\pi i x}\right)\,\mathit{Op}\left(e^{2\pi i y}\right) = e^{\frac{\pi i}{N}}\,\mathit{Op}\left(e^{2\pi i (x+y)}\right).$$

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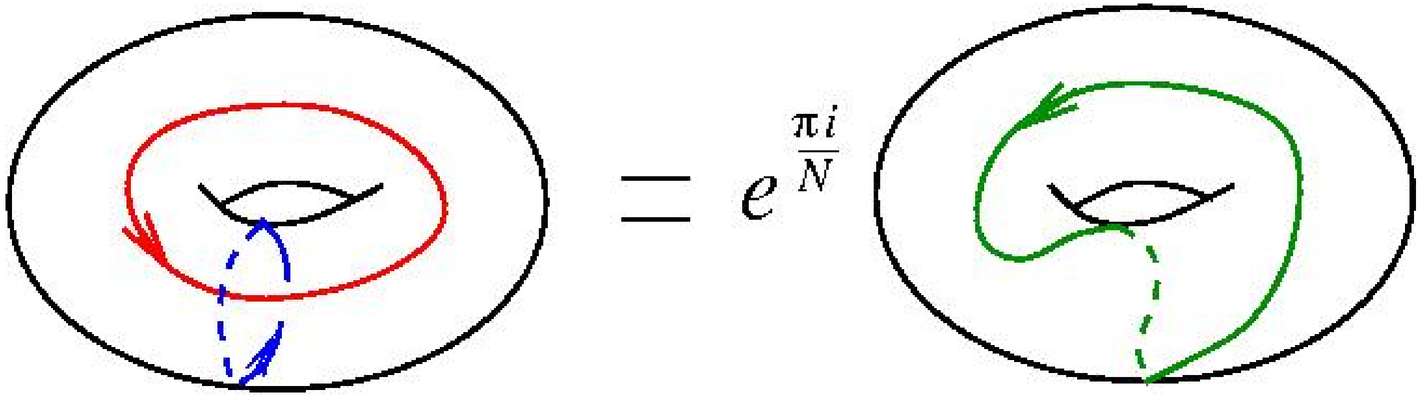

$$Op\left(e^{2\pi i x}\right) Op\left(e^{2\pi i y}\right) = e^{\frac{\pi i}{N}} Op\left(e^{2\pi i (x+y)}\right).$$

So the Heisenberg group is a group of curves!

The representation of the Heisenberg group $\mathbf{H}(\mathbb{Z}_{2N}^{g})$ *on theta functions arises as an induced representation.*

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Extend the character

$$\chi : \{(0,0,k) \,|\, k \in \mathbb{Z}_{2N}\} \to \mathbb{C}, \quad \chi((0,0,k)) = e^{\frac{\pi i}{N} k}$$

to a maximal abelian subgroup of $\mathbf{H}(\mathbb{Z}_{2N}^g)$.

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Example: the subgroup with elements of the form $(0,q,k)$.

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to a maximal abelian subgroup of $\mathbf{H}(\mathbb{Z}_{2N}^g)$*.*

The representation induced by χ *is the left regular action of the Heisenberg group on a quotient of its group algebra by relations of the form* $uu' - \chi(u')^{-1}u = 0$*, for* u' *in the maximal abelian subgroup.*

The representation of the Heisenberg group $\mathbf{H}(\mathbb{Z}_{2N}^g)$ on theta functions arises as an induced representation.

$\mathbf{H}(\mathbb{Z}_{2N}^g)$*: group of curves* $\longrightarrow$ $\mathbb{C}[\mathbf{H}(\mathbb{Z}_{2N}^g)]$*: algebra of curves* $\longrightarrow$ *space of theta functions as a quotient of* $\mathbb{C}[\mathbf{H}(\mathbb{Z}_{2N}^g)]$ *obtained by filling the inside of the surface.*

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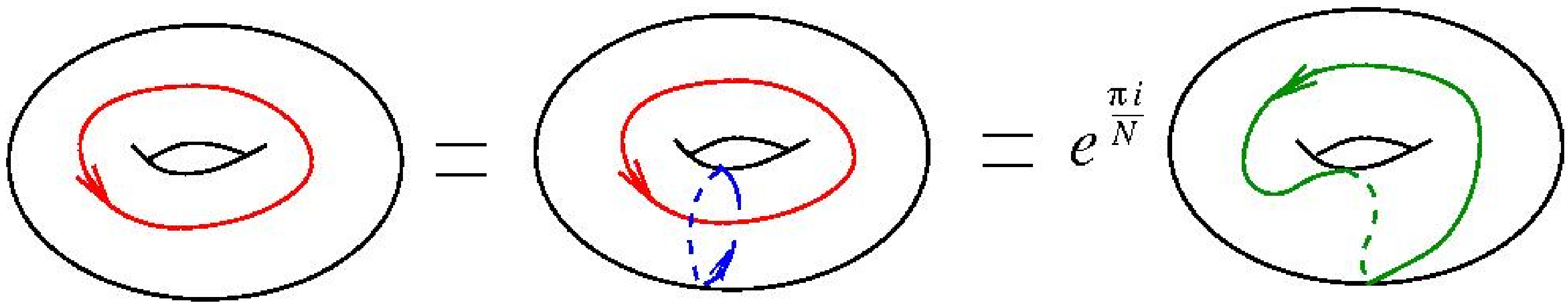
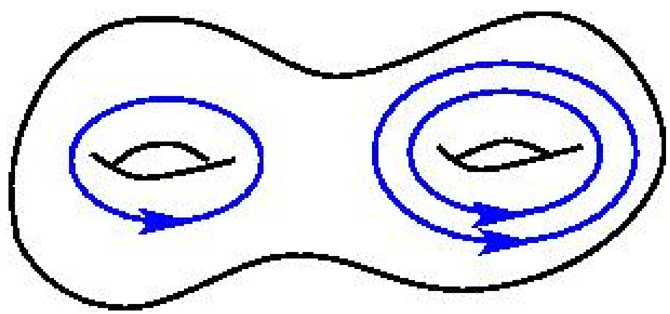


We need to add framing to the curves!

The representation of the Heisenberg group $\mathbf{H}(\mathbb{Z}_{2N}^g)$ *on theta functions arises as an induced representation.*

group of curves $\longrightarrow$ *algebra of curves* $\longrightarrow$ *fill the inside of the surface*

The theta functions $\theta_\mu^\Pi(z)$ *are*



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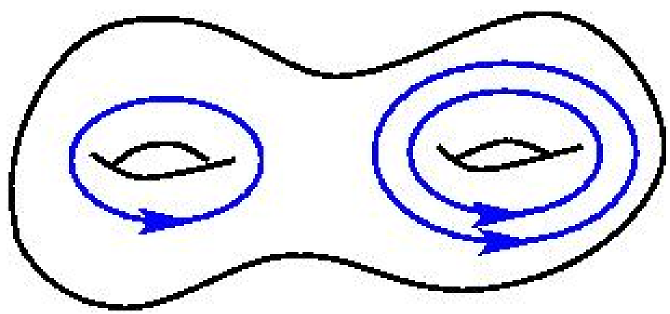

The space of theta functions is a skein module of the handlebody.

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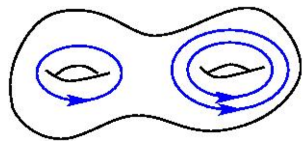

The space of theta functions is a skein module of the handlebody. The notion of a skein module was introduced by J. Przytycki: Factor the space of framed links by skein relations.

The representation of the Heisenberg group on theta functions arises as an induced representation.

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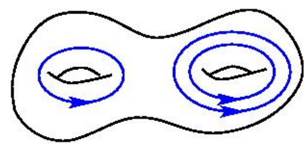
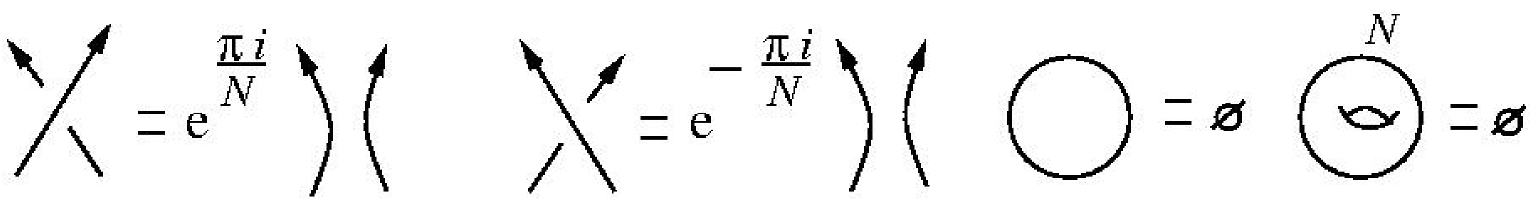


The space of theta functions is a skein module of the handle-body. Factor the vector space with basis the framed links inside the handlebody by the skein relations:



The action of the mapping class group

If h is a homeomorphism of the surface, then h acts linearly on the first homology group, and so it acts on exponentials. Because of the "Stone-von Neumann" theorem there exists an automorphism on the space of theta functions, $\mathcal{F}(h)$, such that

$$\mathit{Op}(h \cdot f) = \mathcal{F}(h)^{-1} \mathit{Op}(f) \mathcal{F}(h).$$

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The map $\mathcal{F}(h)$ is a discrete Fourier transform. The above equation is known in the theory of pseudo-differential operators as an exact Egorov identity.

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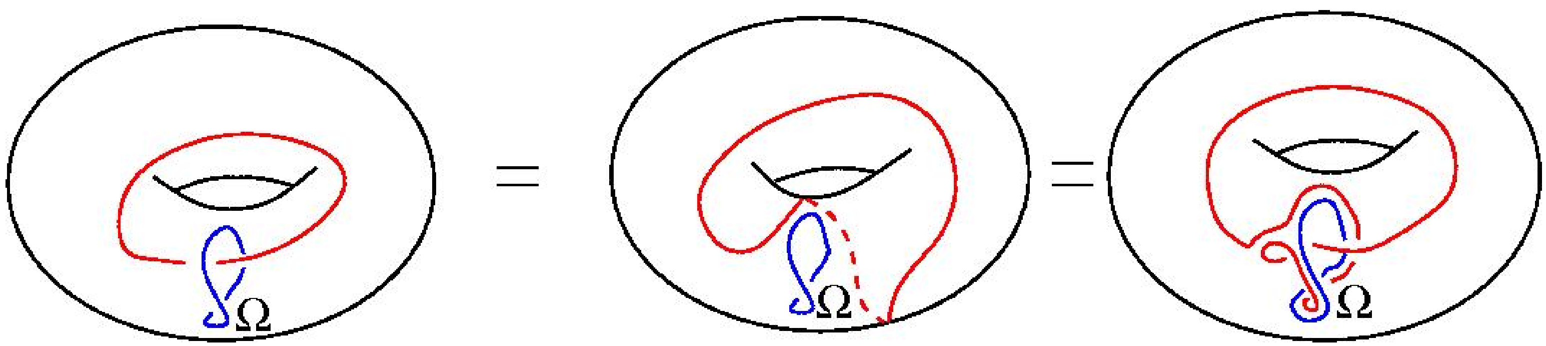
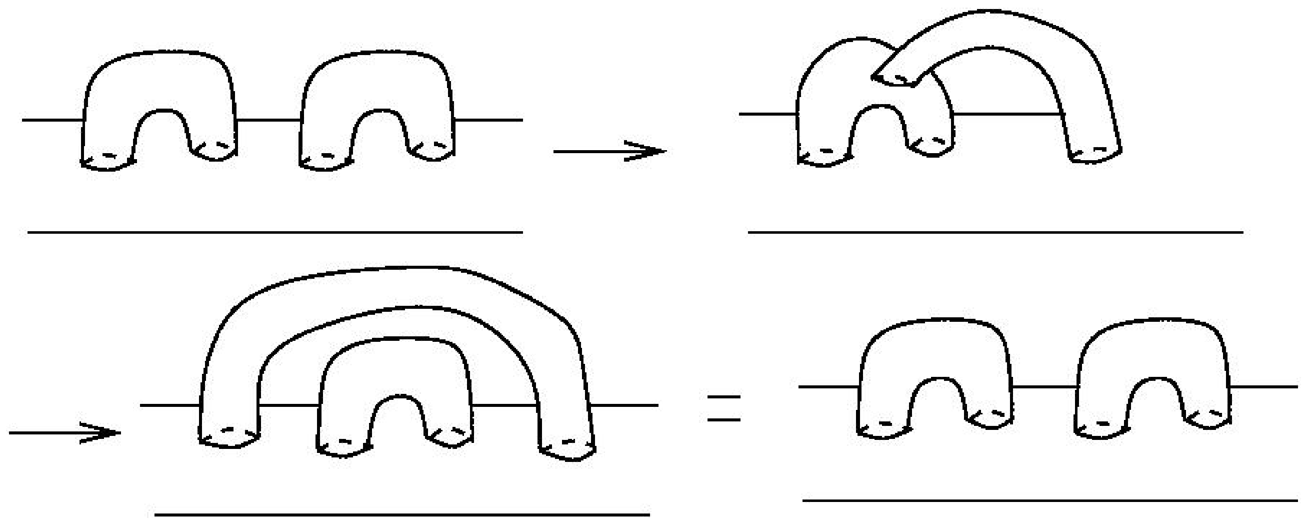
This can be translated into topological language, and interpreted in terms of "handle slides" in dimension 4.



Handleslide in dimension 3 for 1-handles:



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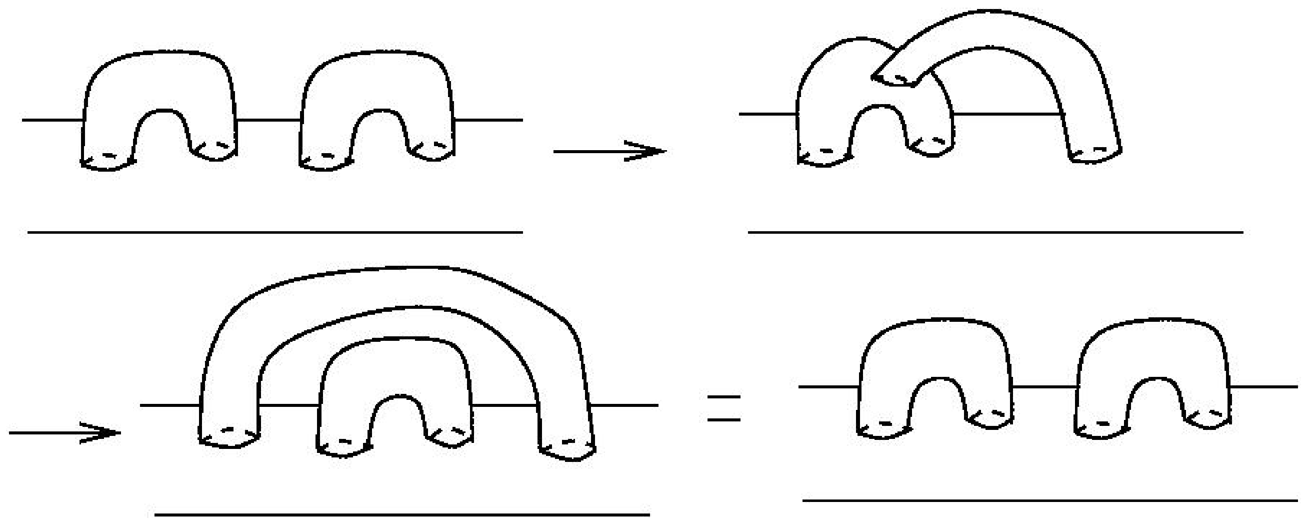
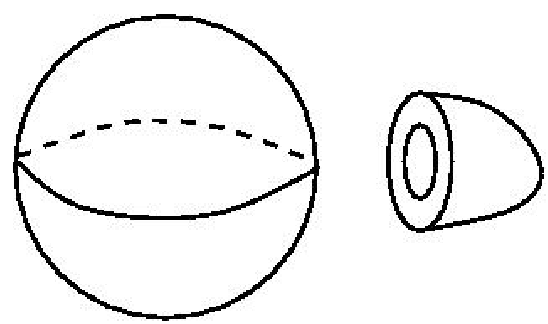


Attaching a 2-handle to a 3-ball:



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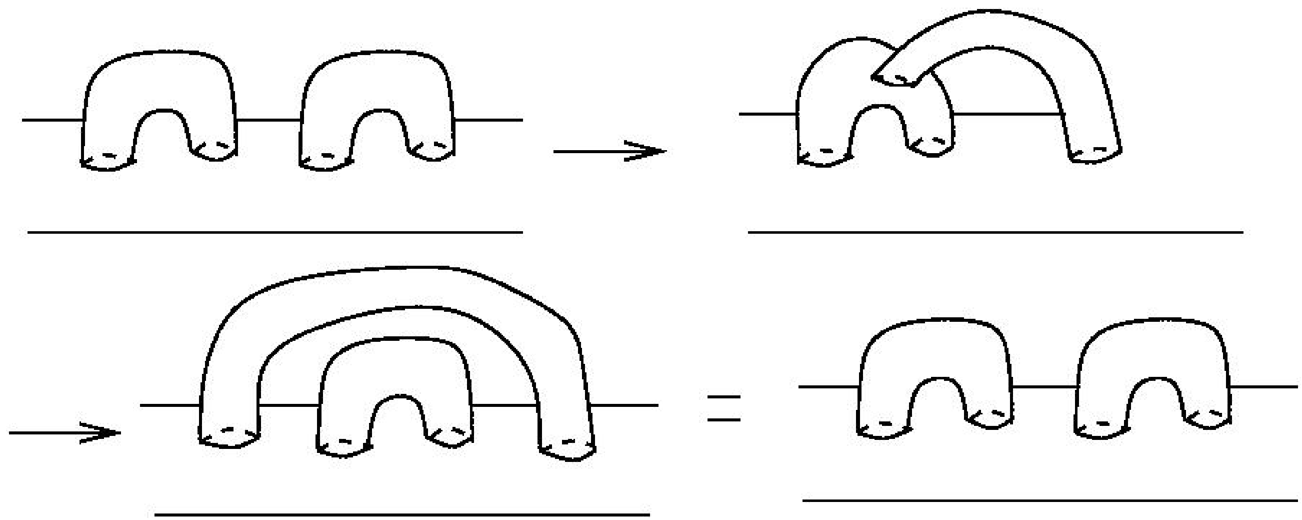
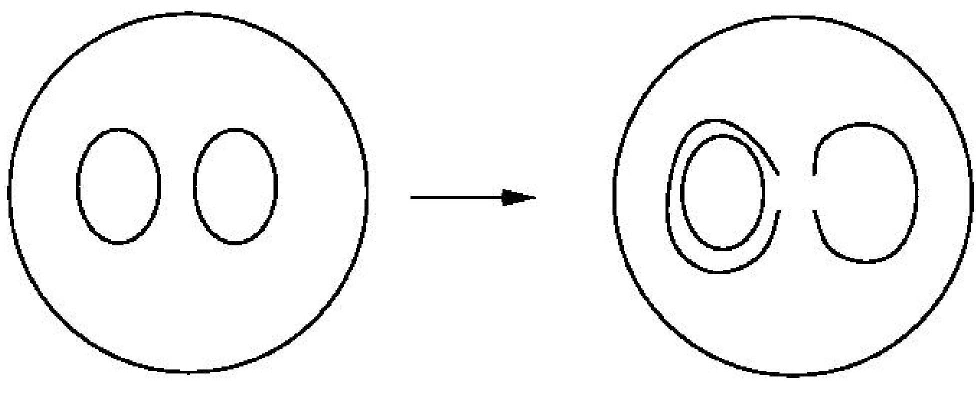


Handleslide in dimension 3 for 2-handles:



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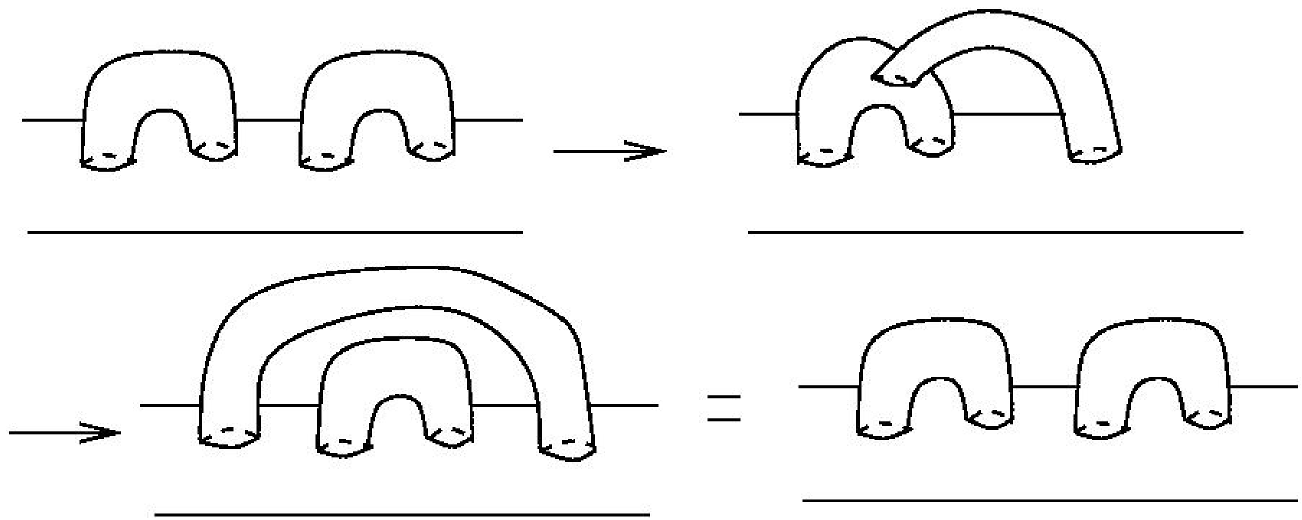
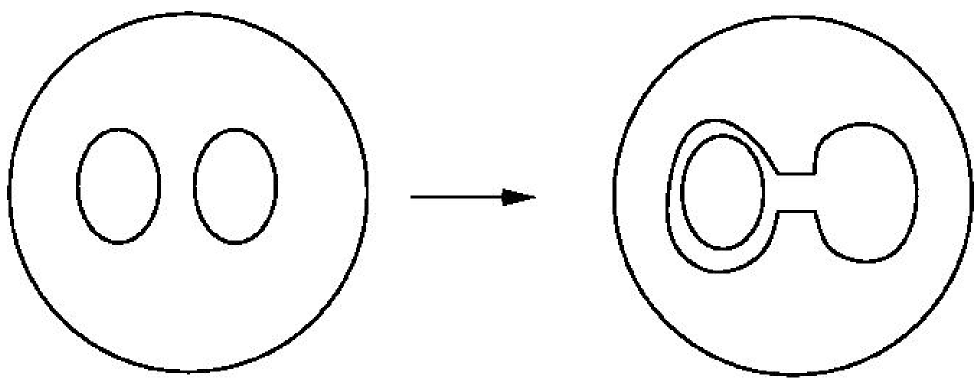


Handleslide in dimension 3 for 2-handles:



Every 3-dimensional manifold can be obtained as the boundary of a 4-dimensional handlebody obtained by attaching 2-handles to a 4-dimensional ball.

Handleslides $\leftrightarrow$ *Kirby calculus*

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Handleslides $\leftrightarrow$ *Kirby calculus*

This yields the 3-manifold invariants and the topological quantum field theory of abelian Chern-Simons theory.

Theorem. (G.-Hamilton) There is a UNIQUE topological quantum field theory that unifies, for Riemann surfaces of all genera, the spaces of theta functions, and the actions of finite Heisenberg groups and modular groups.

Finite Heisenberg group $\longrightarrow$ *2D*

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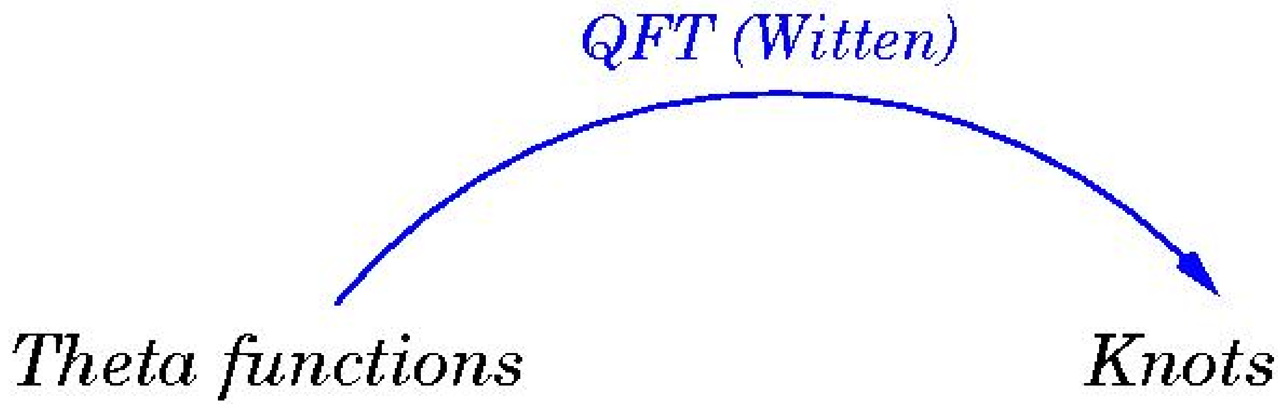
With Hamilton and Uribe we were able to recover the main constructs of Edward Witten's abelian Chern-Simons theory.

Theta functions

Knots


QFT (Witten)
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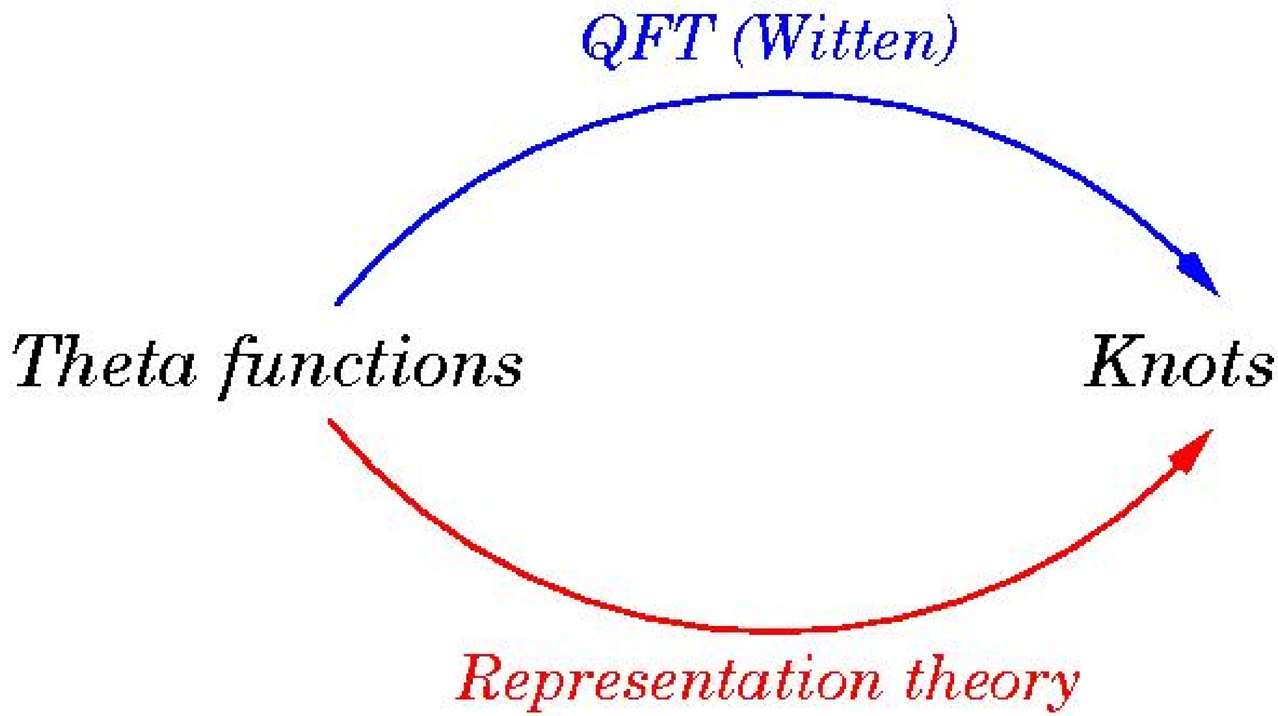
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Representation theory

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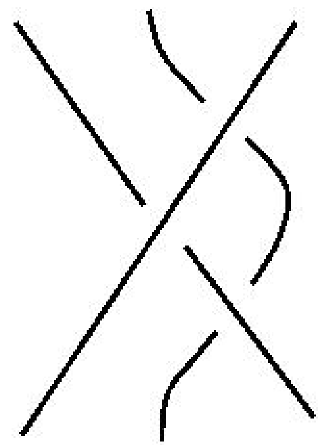

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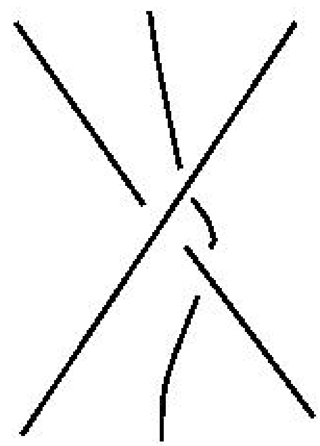

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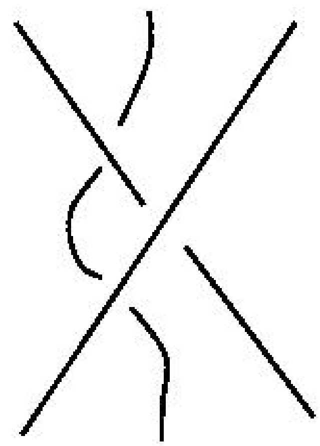

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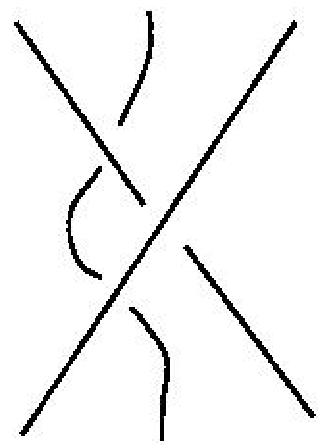

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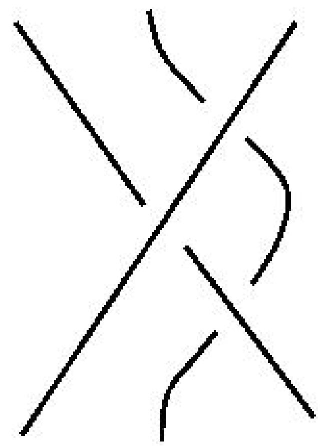

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So there is a quantum group that models the space of theta functions, the action of the finite Heisenberg group, and of the modular group. This quantum group is associated to $U(1)$.

The quantum group associated to theta functions is the Hopf algebra $\mathbb{C}[\mathbb{Z}_{2N}] = \mathbb{C}[K]/K^{2N} = 1$, *with comultiplication, counit and antipode defined by*

$$\Delta(K^j) = K^j \otimes K^j, \quad \epsilon(K^j) = 1, \quad S(K^j) = K^{2N-j}.$$

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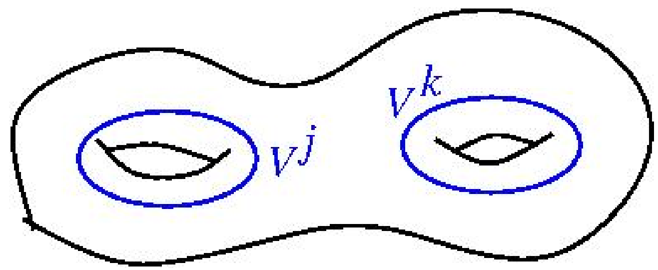
This quantum group is NOT a modular Hopf algebra!

Theta functions are knots and links inside the handlebody colored by representations of this quantum group.



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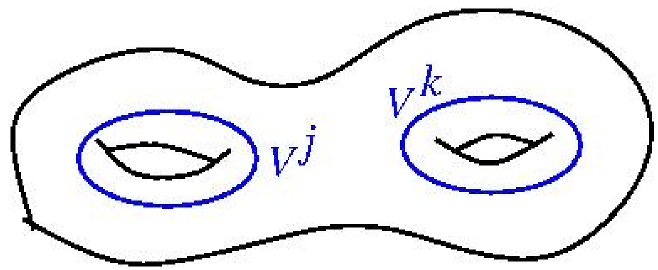

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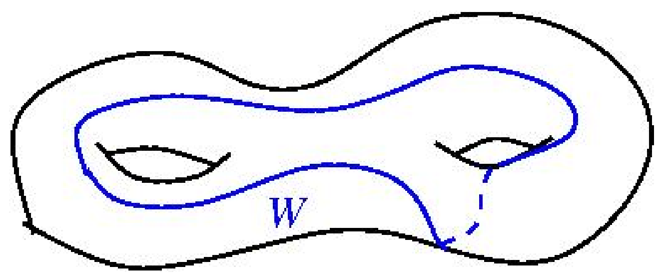

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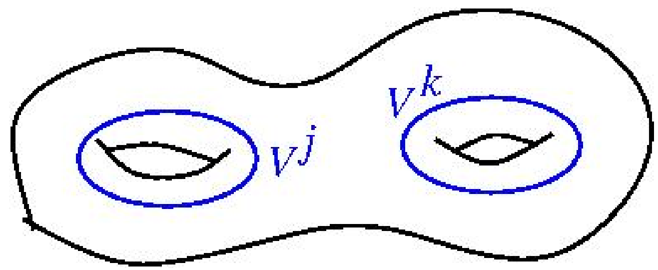

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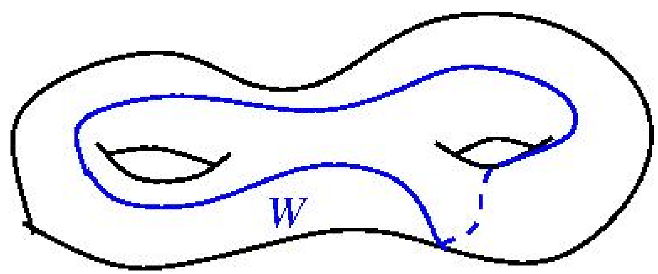

The discrete Fourier transforms of the action of the modular group are represented by certain curves on the boundary colored by elements of the representation ring of the quantum group.