Complex Analysis – Homework 7

1. What conditions on a and b guarantee that $z^a(1-z)^b$ can be defined as a single-valued function on $\mathbb{C}\backslash[0,1]$? In this case describe the Riemann surface of this function?
2. Show that the set of points $\{(z,w)\in\mathbb{C}^2\,|\,w^2=\sin z\}$ is a Riemann surface.
3. Let $P(z)$ be a polynomial with $2n+1$ distinct complex roots. Show that $X = \{(z,w)\,|\,w^2=P(z)\}$ is a Riemann surface. What is its genus?
4. With the notation from the lecture notes, show that for every ω_1 and ω_2 such that $\operatorname{Im}\,\omega_2/\omega_1>0$, there is a τ in the upper half plane such $\mathbb{C}/\Lambda$ and $\mathbb{C}/L$ are conformally equivalent.