Introduction to Topology – Homework 4

1. Show that a finite union of compact subspaces of a topological space X is compact.

2. Let A and B be compact subspaces of X respectively Y, and let N be an open set in $X \times Y$ containing $A \times B$. Show that there exist open sets U and V in X, respectively Y, such that $A \times B \subset U \times V \subset N$.

3. Show that every compact subspace of a metric space is bounded and closed. Is the converse true?

4. Show that the n-dimensional projective space is compact.

5. Is the group $O(n, \mathbb{R})$ of orthogonal matrices with real entries compact? (The topology is defined by the inclusion of $n \times n$ matrices in $\mathbb{R}^{n^2}$.)