Introduction to Topology – Homework 3

1. Give an example of a topological space with 2 connected components and 3 path connected components.
2. Let $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n$ be distinct point in $\mathbb{R}^n$. Show that $\mathbb{R}^n \backslash \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n\}$ is path connected.
3. Show that $\mathbb{R}^2 \backslash \mathbb{Q}^2$ is path connected.
4. If A is path connected does it follow that $\overline{A}$ is path connected?
5. Show that if U is an open connected subspace of $\mathbb{R}^n$, then U is path connected.
6. What are the connected components and the path connected components of $\mathbb{R}^{\mathbb{N}}$ in the box topology?
7. Show that S^1 and S^2 are not homeomorphic.