Introduction to Topology – Homework 1

1. Check that the examples 1, 5, 6 from 1.2 describe indeed topologies.

2. Show that, given two topologies $\mathcal{T}$ and $\mathcal{T}'$ on the set X, the topology $\mathcal{T}$ is finer than $\mathcal{T}'$ if and only if the map $1_X : (X, \mathcal{T}) \to (X, \mathcal{T}')$, $1_X(x) = x$ for all x is continuous.

3. How many different topologies can be defined on a set with four elements?

4. Show that the family

$$\mathcal{B} = \{(a, b) \mid a, b \in \mathbb{Q}\}$$

   is a basis for the standard topology on the real axis.

5. Show that the upper limit topology and the lower limit topology are different. Show that the collection

$$\mathcal{B} = \{[a, b) \mid a < b, a, b \in \mathbb{Q}\}$$

   is a basis for a topology that is different from the lower limit topology. Show that this topology is coarser than the lower limit topology.

6. Prove Proposition 1.3.6.

7. Show that if the topological space Y is a subspace of of the topological space X, and A is a subset of Y, then the topology that A inherits from Y is the same as the topology that it inherits from X.