Introduction to Topology II – Homework 1

1. Classify the covering spaces of the circle. Describe what they are.
2. Show that the only covering spaces of the 2-dimensional torus are tori, cylinders and the plane. What about the n-dimensional case?
3. Let G be a topological group and $p : (\tilde{G}, \tilde{e}) \to (G, e)$ a covering map, where e is the identity element. Show that there is a unique multiplication on $\tilde{G}$ with $\tilde{e}$ the identity element, such that p is a group homomorphism.
4. Show that if the action of the group of deck transformations in one fiber is transitive, then its action in every fiber is transitive.
5. Find the universal covering space of the figure eight. Compute the fundamental group of figure eight.