MATH 5321 Exam II 9 April 2014

Take Home

Due: 16 April

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!**

Each problem is worth 6 points.

1. Using either the Argument Principle or Rouché's Theorem, determine how many roots $e^{2z-1}=\alpha z^3$ has inside $|z|=1$ for $|\alpha|>\pi$.

2. Using either the Argument Principle or Rouché's Theorem, determine how many of the roots of $z^4+7z-1=0$ belong to $\operatorname{ann}(0;1\tfrac{1}{6},2)$.

3. Using either the Argument Principle or Rouché's Theorem, find the number of roots in the first quadrant of $p(z)=z^4+5z^3+6z^2+15z+8$.

4. Using either the Argument Principle or Rouché's Theorem, find the number of roots in the first quadrant of $p(z)=z^4+5z^3+6z^2-15z+8$.

5. Let $D$ denote the open unit disk (centered at 0) and let $UHP$ denote the upper half-plane. Let $F=\{f\in\mathscr{A}(D): f:D\to UHP, f(0)=6i\}$. Find the value of $\alpha=\max_{f\in F}|f'(0)|$.

6. Show that there does not exist a non-constant function $u$ such that $u$ is harmonic on $\mathbb{C}$ and for each $z=x+iy\in\mathbb{C}$ that $u(z)>4x^2+9y^2+1$.

7. Let $\Omega$ and $G$ be regions in $\mathbb{C}$. Let $f\in\mathscr{A}(\Omega)$ such that $f(\Omega)\subset G$ and let $u$ be harmonic on $G$. Show that $u\circ f$ is harmonic on $\Omega$.

8. Let $G$ be a region. Let $u$, $v$ be harmonic on $G$.
   a. Prove that if $u$, $v$ are harmonic conjugates, then the product $uv$ is again harmonic on $G$.
   b. Give an example to show that $u$, $v$ are not harmonic conjugates, then the product $uv$ need not be harmonic on $G$.

9. Consider the following two "theorems". One is true and one is false.

   A. Let $G$ be a region in $\mathbb{C}$ and let $F$ be a normal subset of $\mathscr{A}(G)$. Let $F'=\{f':f\in F\}$. Then, $F'$ is normal.

   B. Let $G$ be a region in $\mathbb{C}$ and let $F$ be a normal subset of $\mathscr{A}(G)$. Let $\grave{F}=\{f:f'\in F\}$. Then, $\grave{F}$ is normal.

   Determine which is true and prove it. Give a counter-example to show that the other is false.