MATH 5321 Exam II 1 April 2009

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!**

1. (30 pts) State and prove Rouché's Theorem.

2. (30 pts) Give the definition for each of the following:

a. Let $f, g \in \mathcal{A}(D)$. Then, f is subordinate to g ($f \prec g$) if ...

b. For $\{K_n\}$ a compact exhaustion of a region G and for $f, g \in \mathcal{C}(G,\Omega)$ let

$\rho(f,g) = \ ...$

c. Let G be a region. A function f is meromorphic on G if ...

d. A set $\mathcal{F} \subset \mathcal{C}(G,\Omega)$ is normal ...

e. Let G be a region and let $f: G \to \mathbb{R}$. f satisfies the Mean Value Property (MVP) on G if ...

f. A set $F \subset \mathcal{C}(G,\Omega)$ is equicontinuous at a point $z_0 \in G$ if ...

3. (20 pts) Find the number of zeros of $p(z) = \frac{11}{10}z^6 - \frac{26}{5}z^5 + 3z^2 - 1$ in $|z| \le 1$.

4. (20 pts) Let G be a bounded region in $\mathbb{C}$. Let $\{f_n\} \subset \mathcal{C}(\overline{G},\mathbb{C}) \cap \mathcal{A}(G)$ and let $f \in \mathcal{C}(\overline{G},\mathbb{C}) \cap \mathcal{A}(G)$. Suppose that $f_n \to f$ uniformly on ∂G. Show that $f_n \to f$ in $\mathcal{C}(G,\mathbb{C})$.