MATH 5320 Exam I October 10, 1997

In-class

Answer the problems on separate paper. You do not need to rewrite the problem statements on your answer sheets. Work carefully. Do your own work. **Show all relevant supporting steps!**

1. Find $(1-i)^{14}$.

2. Suppose $z = -2\bar{z}$. Characterize z.

3. Prove or disprove: $\overline{A \cap B} = \bar{A} \cap \bar{B}$

4. Prove: $\mathbb{C}$ is complete.

5. Prove: Every sequentially compact metric space is complete.

6. Prove: $\mathrm{d}(x,A) = \mathrm{d}(x,\bar{A})$.

7. Definition: A point $z \in A \subset \mathbb{C}$ is said to be *isolated in A* if there exists a ball $B(z,r)$ such that $B(z,r) \cap A = \{z\}$. Prove: If $M = \bigcup_{\lambda \in \Lambda} z_\lambda$ and each z_λ is isolated in M, then M is countable.

8. Prove: If $z,w \in \mathbb{C}$ and $zw = 0$, then either $z = 0$ or $w = 0$.

9. Prove: If $\sum_{n=1}^{\infty} u_n$, is convergent, then there exists an M such that $|u_n + u_{n+1} + u_{n+2} + \cdots + u_{n+p}| < M$, for all n and p.

10. Is $f(z) = f(x+iy) = x - iy$ differentiable anywhere on $\mathbb{C}$? analytic anywhere on $\mathbb{C}$?