Math 5318 Exam I October 6, 2000

In Class Portion:
All outside references must be to definitions and/or theorems in Chapters 1.1 - 2.6 & Appendix.

1. Prove the following proposition: *Proposition. The multiplicative identity for the real number system is unique, i.e., there exists only one real number which satisfies the requirements postulated in the field axioms about the existence of a multiplicative identity.*

2. Prove, from the page *Theorems for Algebra*: Theorem 1.h.iii.

3. Prove from the page *Theorems for Algebra*: Theorem 2.c.iii.

4. Give an example, if one exists, of:

   A. A sequence which is unbounded and which has a convergent subsequence.
   B. A sequence which has three subsequences, each of which converges to a different value.

5. Investigate the convergence of the sequence $\{s_n\}_{n=1}^{\infty}$ where $s_n = \sqrt{n^2 + n} - n$.

6. Consider the sequence $\{s_n\}$ given by $s_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!}$. Prove $\{s_n\}_{n=1}^{\infty}$ converges.

7. Prove the following theorem: *Theorem. Every non-increasing sequence which is bounded below is convergent.*

Take Home Portion (Due Tuesday, Noon):
All outside references must be to definitions and/or theorems in Chapters 1.1 - 2.6 & Appendix.

1. Prove that the sequence $\{s_n\}_{n=1}^{\infty}$ given by $s_n = \dfrac{(n^2 + 20n + 35)\sin(n^3)}{n^2 + n + 1}$ has a convergent subsequence.

2. Let $0 < s_1 \leq 3$ and $s_{n+1} = \sqrt{2s_n + 3}$ for $n \in I$. Prove $s_n \to 3$.

3. Let $0 < s_1 < 1$ and $s_{n+1} = 1 - \sqrt{1 - s_n}$ for $n \in I$. Prove $\{s_n\}_{n=1}^{\infty}$ converges.

4. Give an example of a family of sets E_t, $t \in \mathbb{R}$, such that: (1) each $E_t \subset \mathbb{R}$, (2) whenever $t_1 < t_2$ then E_{t_1} is a proper subset of E_{t_2} and (3) $\bigcup_{t \in \mathbb{R}} E_t$ is countable.