MATH 3360 Exam I February 15, 1995

Show your work for each problem. You do <u>not</u> need to rewrite the statements of the problems on your answer sheets.

1. Assume that $\alpha : S \to T$ and $\beta : T \to U$. Consider the following statement:

$$\text{If } \beta \text{ is not onto, then } \beta \circ \alpha \text{ is not invertible.} \qquad (1)$$

(a) Is statement (1) true? Why or why not?
(b) State the converse of statement (1).
(c) Is the converse of statement (1) true? Why or why not.

2. Let E be the set of even integers. Let $m * n = \dfrac{m \cdot n}{2}$.
(a) Show that * is an operation on E.
(b) Is * associative? Why or why not?
(c) Show that * has an identity.

3. Is $(G,*)$ a group? Why or why not?

(a) $G = \{ e^n \mid n \in \mathbb{Z} \}$; * is multiplication.
(b) $G = \mathbb{Z} \cup \{ z \mid 1/z \in \mathbb{Z} \} \cup \{ 0 \}$; * is addition.

4. Write $(2\ 3\ 5)(1\ 2\ 5\ 3)(1\ 3\ 4)$ as a single cycle or a product of pairwise disjoint cycles.

5. Complete: If G with operation * is a group, then G is non-Abelian iff $a*b \neq b*a$ . . . .

6. Show that S_4 is non-Abelian.

7. It can be shown that $\mathbb{R}^2$ with operation + being component-wise addition, i.e., $(a,b) + (c,d) = (a+c,b+d)$, is a group, . Let $L = \{ (x,y) \mid 2x - 3y = 0 \}$, i.e., L is the set of points which lie on the line satisfying the equation $2x - 3y = 0$. Show that $(L,+)$ is a subgroup of $(\mathbb{R}^2,+)$.

8. Identify the symmetry group for the following figure, which consists of a square with one diagonal and the middle third of the other diagonal.