Find the indicated integrals in Problems 21–36.

21. $\int \frac{2x^3 + 9x - 1}{x^2(x^2 - 1)}\,dx$
22. $\int \frac{x^4 - x^2 + 2}{x^2(x - 1)}\,dx$
23. $\int \frac{x^2 + 1}{x^2 + x - 2}\,dx$
24. $\int \frac{dx}{x^3 - 8}$
25. $\int \frac{x^4 + 1}{x^4 - 1}\,dx$
26. $\int \frac{x^3 + 1}{x^3 - 1}\,dx$
27. $\int \frac{x\,dx}{(x + 1)^2}$
28. $\int \frac{2x\,dx}{(x - 2)^2}$
29. $\int \frac{dx}{x(x + 1)(x - 2)}$
30. $\int \frac{x + 2}{x(x - 1)^2}\,dx$
31. $\int \frac{x\,dx}{(x + 1)(x + 2)^2}$
32. $\int \frac{x + 1}{x(x^2 + 2)}\,dx$
33. $\int \frac{5x + 7}{x^2 + 2x - 3}\,dx$
34. $\int \frac{5x\,dx}{x^2 - 6x + 9}$
35. $\int \frac{3x^2 - 2x + 4}{x^3 - x^2 + 4x - 4}\,dx$
36. $\int \frac{3x^2 + 4x + 1}{x^3 + 2x^2 + x - 2}\,dx$

B 37. **WHAT DOES THIS SAY?** Describe the process of partial fraction decomposition.

Find the indicated integrals in Problems 38–55.

38. $\int \frac{\cos x\,dx}{\sin^2 x - \sin x - 2}$
39. $\int \frac{e^x\,dx}{2e^{2x} - 5e^x - 3}$
40. $\int \frac{e^x\,dx}{e^{2x} - 1}$
41. $\int \frac{\sin x\,dx}{(1 + \cos x)^2}$
42. $\int \frac{\tan x\,dx}{\sec^2 x + 4}$
43. $\int \frac{\sec^2 x\,dx}{\tan x + 4}$
44. $\int \frac{dx}{x^{1/4} - x}$
45. $\int \frac{dx}{x^{2/3} - x^{1/2}}$
46. $\int \frac{dx}{\sin x - \cos x}$
47. $\int \frac{dx}{3\cos x + 4\sin x}$
48. $\int \frac{dx}{5\sin x + 4}$
49. $\int \frac{\sin x - \cos x}{\sin x + \cos x}\,dx$
50. $\int \frac{dx}{4\cos x + 5}$
51. $\int \frac{dx}{\sec x - \tan x}$
52. $\int \frac{dx}{3\sin x + 4\cos x + 5}$
53. $\int \frac{dx}{4\sin x - 3\cos x - 5}$
54. $\int \frac{dx}{2\csc x - \cot x + 2}$
55. $\int \frac{dx}{x(3 - \ln x)(1 - \ln x)}$

56. Find the area under the curve $y = \frac{1}{x^2 + 5x + 4}$ between $x = 0$ and $x = 3$.

57. Find the area of the region bounded by the curve $y = \frac{1}{6 - 5x + x^2}$ and the lines $x = \frac{4}{3}$, $x = \frac{7}{4}$, and $y = 0$.

58. Find the volume (to four decimal places) of the solid generated when the curve

$$y = \frac{1}{x^2 + 5x + 4}, \qquad 0 \le x \le 1$$

is revolved about

a. the y-axis **b.** the x-axis **c.** the line $x = -1$

59. Find the volume (to four decimal places) of the solid generated when the region under the curve

$$y = \frac{1}{\sqrt{x^2 + 4x + 3}}$$

on the interval $[0, 3]$ is revolved about

a. the x-axis **b.** the y-axis

60. Find both the exact and approximate volume of the solid generated when the region under the curve $y^2 = x^2\left(\frac{4 - x}{4 + x}\right)$ on the interval $[0, 4]$ is revolved about the x-axis.

61. HISTORICAL QUEST George Plya was born in Hungary and attended universities in Budapest, Vienna, Gttingen, and Paris. He was a professor of mathematics at Stanford University. Plya's research and winning personality earned him a place of honor not only among mathematicians, but among students and teachers as well. His discoveries spanned an impressive range of mathematics, including real and complex analysis, probability, combinatorics, number theory, and geometry. Plya's book, *How to Solve It*, has been translated into 20 languages. His books have a clarity and elegance seldom seen in mathematics, making them a joy to read. For example, here is his explanation of why he was a mathematician: "It is a little shortened but not quite wrong to say: I thought I am not good enough for physics and I am too good for philosophy. Mathematics is in between." ■

GEORGE PÓLYA
1887–1985

A story told by Pólya provides our next Quest. "A number of years ago," Pólya related with his lovable accent, "I deliberately put the problem

$$\int \frac{x\,dx}{x^2 - 9}$$

as the first problem on a test of techniques of integration, to give my students a boost as they began the exam. With the substitution $u = x^2 - 9$, which I expected the students to use, you can knock the problem off in just a few seconds. Half of the students did this, and got off to a good start. But a fourth of them used the correct but time-consuming procedure of partial fractions—and because they spent so much time on the problem, they did poorly on the exam. Half of the rest used the trig substitution $x = 3\sin\theta$—also correct but so time-consuming that they wound up very far behind and bombed the exam. It is interesting that the students who used the harder techniques showed they knew 'more,' or at least more difficult mathematics than the ones who used the easy technique. But they showed that 'it's not just what you know; it's how and when you use it.' It's nice when what you do is right, but it's much better when it's also *appropriate*."* Carry out all three methods of solution of the given integral Pólya described in this quotation.

*"Pólya, Problem Solving, and Education," by Alan H. Schoenfeld, *Mathematics Magazine*, Vol. 60, No. 5, December 1987, p. 290.