Section 5.2/5.3

I. Summary: Algebra, Limits, Calculus → Differential, Integral

II. Integral Calculus

A. Particular Example (Uniform Partition): Computation of Area as a Sum
B. General Setting: Construction of the Riemann Integral

III. General Scheme for $y = f(x)$ on the Interval $[a,b]$

A. Partition the Interval

Uniform Partition ($\| P \| = \dfrac{b-a}{n}$)

$$P = \{a = x_0 < a + \Delta x = x_1 < a + 2\Delta x = x_2 < a + 3\Delta x = x_3 < \cdots a + n\Delta x = x_n = b\}$$
$$\Delta x = \frac{b-a}{n}$$

General Setting ($\| P \| = \max\limits_{1 \le k \le n} \Delta x_k$)

$$P = \{a = x_0 < x_1 < x_2 < \cdots < x_n = b\}$$
$$\Delta x_k = x_k - x_{k-1}$$

B. Choose a Distinguished Point x_k^* in each Subinterval of the Partition

Uniform Partition $\quad x_k^* = a + k\Delta x$

General Setting $\quad x_k^* \in [x_{k-1}, x_k]$

C. Compute Function Values $f(x_k^*)$ over the Partition

D. Form the <u>*Riemann Sum*</u> of the Products $f(x_k^*)\Delta x_k$ over the Partition

Uniform Partition

$$f(a+\Delta x)\Delta x + f(a+2\Delta x)\Delta x + f(a+3\Delta x)\Delta x + \cdots + f(a+n\Delta x)\Delta x$$

General Setting

$$f(x_1^*)\Delta x_1 + f(x_2^*)\Delta x_2 + f(x_3^*)\Delta x_3 + \cdots + f(x_n^*)\Delta x_n$$

Examples (Graphical)

IV. Summation Notation

A. Basic Properties

1. Constant Term
2. Sum Rule
3. Scalar Multiple Rule
4. Linearity Rule
5. Subtotal Rule
6. Dominance Rule

1. Formulas

2. Examples

V. Riemann Integral for $y = f(x)$ on the Interval $[a,b]$

A. Form the Riemann Sum as Described in Step III Above

B. Compute the Limit of the Riemann Sum as $\| P \| \to 0$

1. ***<u>If</u>*** the limit exists, we say the function f is <u>*integrable*</u> on the interval $[a,b]$

2. We denote the limit by $\int_a^b f(x)dx$ and call it the <u>*definite integral*</u> of f from *a* to *b*

3. The function f is called the <u>*integrand*</u>; the interval $[a,b]$ is called the <u>*interval of integration*</u>; the endpoints *a* and *b* are the called the <u>*lower and upper limits of integration*</u>, resp.

4. Theorem (Advanced Calculus) *If f is continuous on an interval $[a,b]$, then f is integrable on $[a,b]$.*

5. If f is continuous on $[a,b]$ and if $f(x) \geq 0$ on $[a,b]$, then $\int_a^b f(x)dx$ = area under the curve $y = f(x)$ over the interval $[a,b]$.

Examples

C. Properties of the Definite Integral

1. Integral at a Point
2. Interchanging the Limits of Integration
3. Linearity
4. Subdivision Rule
5. Dominance

Examples