

Section 5.1

I. Anti-derivative: Definition

II. General Anti-derivative

III. Notation: $\int f(x) dx = F(x) + C$

A. Language: Anti-derivative == Indefinite Integral
Anti-differentiation == Indefinite Integration

IV. Basic Formulas

Differentiation	Integration
$\frac{d(cf(x))}{dx} = c \frac{df(x)}{dx}$	$\int cf(x) dx = c \int f(x) dx$
$\frac{d}{dx}(f(x) + g(x)) = \frac{df(x)}{dx} + \frac{dg(x)}{dx}$	$\int (f(x) + g(x)) dx = \int f(x) dx + \int g(x) dx$

V. Basic Rules

Differentiation	Integration
$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln x + C$
$\frac{d}{dx}(e^x) = e^x$	$\int e^x dx = e^x + C$

Comments

VI. Basic Rules for Trigonometric Functions

Differentiation	Integration
$\frac{d}{dx}(\sin x) = \cos x$	$\int \cos x \, dx = \sin x + C$
$\frac{d}{dx}(\cos x) = -\sin x$	$\int \sin x \, dx = -\cos x + C$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec^2 x \, dx = \tan x + C$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\int \sec x \tan x \, dx = \sec x + C$

VII. Examples

A. Direct and Indirect

$$\begin{array}{lll}
 \int (2x^3 - 5x + 7\sqrt{x}) \, dx & \int \frac{x^3 - 10\sqrt{x} + 3}{x^2} \, dx & \int (2x^2 + 3)^2 \, dx \\
 \int \frac{x^2 - 10\sqrt{x+3}}{x^3} \, dx & \int \frac{t^3 - 10\sqrt{t} + 3}{t^2} \, dt & \int \left(1 + x^2 + \frac{1}{1+x^2}\right) \, dx
 \end{array}$$

B. Notational Impact

$$\int (2x+1)^5 \, dx \qquad \int \cos x^2 \, dx \qquad \int \frac{3}{1-x} \, dx \qquad \int e^{-3x} \, dx$$

VIII. Area as an Anti-derivative