Section 4.5

I. Indeterminate Forms

A. $0/0 \quad \infty/\infty$

B. $1^{\infty} \quad 0 \bullet \infty \quad 0^0 \quad \infty^0 \quad \infty - \infty$

Examples

$$\lim_{x\to 0}\frac{\sin 4x}{3x} \qquad \lim_{x\to 0}\frac{x\sin x}{1-\cos x} \qquad \lim_{x\to 2}\frac{x^2-3x+2}{x^2-4} \qquad \lim_{x\to 0}\frac{1-e^{-x}}{\sin x}$$

$$\lim_{x\to \infty}\frac{\ln x}{x} \qquad \lim_{x\to \infty}\frac{x}{e^x} \qquad \lim_{x\to \infty}\frac{x^2-3x+2}{x^2-4} \qquad \lim_{x\to \infty}\left(1+\frac{1}{x}\right)^{3x}$$

$$\lim_{x\to 0^+} x\ln x \qquad \lim_{x\to 0^+}\left(x^2+x\right)^{\sin 3x} \qquad \lim_{x\to 0^+}\left(\ln x\right)^{\tan x} \qquad \lim_{x\to \infty}\sqrt{x^2+4x}-x$$

Non-Examples

$$\lim_{x\to 0}\frac{\tan x}{\cos^2 x} \qquad \lim_{x\to 0^+}\frac{x^2+x+1}{3x^2+x} \qquad \lim_{x\to \infty} x-\sin x \qquad \lim_{x\to \infty} x\sin x$$

$$\lim_{x\to \infty}\frac{\sin 2x}{e^x} \qquad \lim_{x\to \infty} x\tan^{-1} x \qquad \lim_{x\to \pi}\frac{1-\cos x}{x}$$

II. L'Hôpital's Rule

A. Theorem *Let I be an open interval and let $c \in I$. Let f and g be differentiable on I such that $g'(x) \neq 0$ for $x \neq c$. Suppose that either*

(*i*) $f(c) = g(c) = 0$

or

(*ii*) $\lim_{x\to c} f(x) = \lim_{x\to c} g(x) = \infty$

Then,

$$\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$$

if the latter limit exists.

Additional Examples

$$\lim_{x\to 0}\frac{x-\sin x}{x^3} \qquad \lim_{x\to 0}\frac{x-\tan x}{\sin x - x} \qquad \lim_{x\to\infty} xe^{-3x} \qquad \lim_{x\to 0}\frac{1}{\sin x}-\frac{1}{x}$$

$$\lim_{x\to 0}\frac{e^x-1-x}{x^2} \qquad \lim_{x\to 1}\frac{x^6-2x^5+2x^3-x^2}{x^5-3x^4+4x^3-4x^2+3x-1}$$

$$\lim_{x\to\infty}\frac{x^n}{e^{ax}},\ n>0, a>0 \qquad \lim_{x\to\infty}\frac{\ln x}{x^a},\ a>0$$