MATH 5317 Introduction to Modern Algebra
Spring 2023
Homework 5 - Due In Class Friday April 28th

1. Let R be a ring. Prove that $R[x]$ is a ring.

2. For a ring R, prove that a $R[x]$ has zero divisors if and only if R has zero divisors.

3. Determine all units in the following polynomial rings:

- **(a)** $\mathbb{Z}[x]$;
- **(b)** $\mathbb{Q}[x]$;
- **(c)** $\mathbb{Z}_7[x]$.

4. Find a zero divisor, if possible, in the following polynomial rings:

- **(a)** $\mathbb{Z}[x]$;
- **(b)** $\mathbb{Z}_4[x]$;
- **(c)** $\mathbb{Z}_7[x]$.

5. Let $f(x) = x^6 + 3x^5 + 3x^4 + x^3 + 3x^2 + 3x + 1$ and $g(x) = x^3 + 4$ be polynomials in $\mathbb{Z}_5[x]$.

- **(a)** Find **the** greatest common divisor of $f(x)$ and $g(x)$ in $\mathbb{Z}_5[x]$.
- **(b)** Find polynomials $u(x)$ and $v(x)$ also in $\mathbb{Z}_5[x]$ such that $\gcd(f(x), g(x)) = u(x)f(x) + v(x)g(x)$.

6. Find the remainder on dividing $4x^4 + 3x^3 + 2x^2 + x$ by $x - \frac{1}{2}$ in $\mathbb{Q}[x]$

7. Let I be the set $I = \{f(x) \in \mathbb{Q}[x] \mid f(i) = f(-i) = 0\}$ where $i = \sqrt{-1}$.

- **(a)** Prove that I is an ideal of $\mathbb{Q}[x]$.
- **(b)** Find a polynomial $g(x) \in \mathbb{Q}[x]$ such that $I =< g(x) >$.