Math 4363 - Combinatorics
Homework 5
Due in Class - Thursday 18 April 2019

1. Let $f(n)$ denote the Fibonacci sequence. By evaluating each of the following expressions for small values of n, conjecture a general formula and then prove it using induction and the Fibonacci recurrence.

(a) $f(1)+f(3)+f(5)+\cdots+f(2n-1)$

(b) $f(0)+f(2)+f(4)+\cdots+f(2n)$

(c) $f(0)-f(1)+f(2)-\cdots+(-1)^n f(n)$

(d) $f(0)^2+f(1)^2+f(2)^2+\cdots+f(n)^2$

2. By examining the Fibonacci sequence, make and prove a conjecture about when $f(n)$ is divisible by 7.

3. Let $h(n)$ be the number of different ways in which the squares of 1-by-n board can be colored, using the colors red, white and blue, so that no two squares that are colored red are adjacent. Find and verify a recurrence relation for $h(n)$. Then find a formula for $h(n)$.

4. Determine the generating function for each of the following sequences,

(a) $1, c, c^2, c^3, \cdots, c^n, \cdots$

(b) $1, -1, 1, -1, \cdots, (-1)^n, \cdots$

(c) $1, \frac{1}{1!}, \frac{1}{2!}, \frac{1}{3!}, \cdots, \frac{1}{n!}, \cdots$

(d) $1, -\frac{1}{1!}, \frac{1}{2!}, -\frac{1}{3!}, \cdots, (-1)^n\frac{1}{n!}, \cdots$

5. Determine the generating function for the sequence $h(n)$ of the number of ways to choose n pieces of fruit from apples, bananas, pears and oranges such that the number of

- apples is even;
- bananas is a multiple of 3;
- oranges is at most 2; and
- pears is at most 1.

Then find a formula for $h(n)$ from the generating function.