
Math 4363 - Combinatorics
Homework 4
Due in Class - Thursday 7 March 2019

1. There are 100 people at a party. Each person has an even number (possible zero) of acquaintances at the party. Prove that there are at least three people at the party with the same number of acquaintances.
2. What is the coefficient of x^5y^{13} in the expansion of $(3x - 2y)^{18}$?
3. Let n be a positive integer. Prove that

$$\sum_{k=0}^n (-1)^k \binom{n}{k}^2 = \begin{cases} 0 & \text{if } n \text{ is odd} \\ (-1)^m \binom{2m}{m} & \text{if } n = 2m. \end{cases}$$

Hint: For $n = 2m$, consider the coefficient of x^n in $(1 - x^2)^n = (1 + x)^n(1 - x)^n$.

4. Find a single binomial coefficient equal to

$$\binom{n}{k} + 3\binom{n}{k-1} + 3\binom{n}{k-2} + \binom{n}{k-3}.$$

5. Prove that, for all real numbers r and all integers m and k ,

$$\binom{r}{m} \binom{m}{k} = \binom{r}{k} \binom{r-k}{m-k}.$$