
Math 4362 - Number Theory
Homework 8
Due in Class - Thursday November 21, 2019

1. (50 points) Evaluate the following Legendre symbols:

(a) $\left(\frac{317}{727}\right)$

(b) $\left(\frac{19}{331}\right)$

(c) $\left(\frac{43}{491}\right)$

(d) $\left(\frac{1015}{1019}\right)$

(e) $\left(\frac{285}{293}\right)$

2. (20 points) For p an odd prime, prove that

$$\left(\frac{7}{p}\right) = \begin{cases} +1 & \text{if } p \equiv 1, 3, 9, 19, 25, 27 \pmod{28}, \\ -1 & \text{if } p \equiv 5, 11, 13, 15, 17, 23 \pmod{28}. \end{cases}$$

3. (30 points) Solve each of the following congruences:

(a) $x^2 \equiv 7 \pmod{3^3}$

(b) $x^2 \equiv 14 \pmod{5^3}$

(c) $x^2 \equiv 2 \pmod{7^3}$