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**Math 4362 - Number Theory**  
**Homework 7**  
**Due in Class - Thursday November 13, 2019**

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1. Show that 3 is a quadratic residue of 23 but a quadratic non-residue of 31.
2. Using the fact that 2 is a primitive root of 19, find all quadratic residues of 19.
3. Using Euler's Criterion, prove that if  $p = 2^k + 1$  is prime then every quadratic non-residue of  $p$  is a primitive root of  $p$ .
4. Calculate the values of the following Legendre symbols:
  - (a)  $\left(\frac{19}{23}\right)$
  - (b)  $\left(\frac{20}{31}\right)$
  - (c)  $\left(\frac{-72}{31}\right)$
5. Use Gauss' Lemma to calculate the values of the following Legendre symbols:
  - (a)  $\left(\frac{8}{11}\right)$
  - (b)  $\left(\frac{7}{13}\right)$
  - (c)  $\left(\frac{5}{19}\right)$
6. For an odd prime  $p$ , prove there are  $\frac{p-1}{2} - \phi(p-1)$  quadratic non-residues of  $p$  that are not primitive roots of  $p$ .