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**Math 4362 - Number Theory**  
**Homework 6**  
**Due in Class - Thursday October 24, 2019**

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1. Find the order of 2
  - (a) modulo 17.
  - (b) modulo 23.
  
2. Find the order of all appropriate positive integers modulo 14, and identify the primitive roots, if any.
  
3. Prove that
  - (a) if  $a$  has order  $hk$  modulo  $n$ , then  $a^h$  has order  $k$  modulo  $n$ .
  - (b) if  $a$  has order  $2k$  modulo an odd prime  $p$ , then  $a^k \equiv -1 \pmod{p}$ .
  
4. Let  $r$  be a primitive root of the positive integer  $n$ . Prove that  $r^k$  is a primitive root of  $n$  if and only if  $\gcd(k, \phi(n)) = 1$ .
  
5. Determine all primitive roots of  $p = 19$ .
  
6. Given that 3 is a primitive root of 43, find the following:
  - (a) all positive integers less than 43 that have order 6 modulo 43;
  - (b) all positive integers less than 43 that have order 21 modulo 43; and
  - (c) all other primitive roots of 43.