Math 4362 - Number Theory Homework 5

Due in Class - Thursday October 18, 2018

- 1. Calculate
 - (a) $\phi(5040)$; and
 - **(b)** $\phi(3456)$.
- **2.** Prove that
 - (a) If $d \mid n$, then $\phi(d) | \phi(n)$;
 - **(b)** If *n* is odd, then $\phi(2n) = \phi(n)$; and
 - (c) If *n* is even, then $\phi(2n) = 2\phi(n)$.
- **3.** Find all solutions of $\phi(n) = 24$.
- **4.** For positive integers m and n prove that
 - (a) $\phi(m)\phi(n) = \phi(mn)\phi(d)/d$, where $d = \gcd(m,n)$
 - **(b)** $\phi(m)\phi(n) = \phi(\gcd(m,n))\phi(\operatorname{lcm}(m,n)).$
- 5. Use Euler's Theorem to
 - (a) evaluate 2^{100000} modulo 77; and
 - (b) find the units digit of 3^{100} .