Problem 12.5 # 17, to be turned in with your homework

Find $\iiint_D xyz\ dV$ where D is the tetrahedron with vertices $(0,0,0)$, $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$.

As you may recall the tetrahedron is the region determined by $0 \leq x \leq 1$, $0 \leq y \leq 1-x$, $0 \leq z \leq 1-x-y$.

I found a slick way of doing this problem, please try it and turn it in with your homework. It is based on the observation that having $1-x$ as your upper limit for the integral with respect to y it is better to expand $(1-x-y)^2 = (1-x)^2 - 2(1-x)y + y^2$. Doing this, when you integrate $xy(1-x-y)^2 = xy(1-x)^2 - 2x(1-x)y^2 + xy^3$ with respect to y and evaluate y at the limits of integration the result is $\frac{1}{24}x(1-x)^4$. Then, the next trick is not to expand the latter but use integration by parts with $u = x$ and $dv = (1-x)^4 dx$.

You must include a graph of the tetrahedron with your solution.