What you must know for the test

Note. The Exam will consist of a problem from each of the following parts (five problems in total). Each problem will be worth 10 points.

Part I. Vectors:

(1) Lenght of a vector $\vec{v} = a\vec{i} + b\vec{j} + c\vec{k} = $?

(2) Unit vector in the direction of the vector $\vec{v} = a\vec{i} + b\vec{j} + c\vec{k} = $?

(3) Given two points $P(x_0, y_0, z_0)$ and $Q(x_1, y_1, z_1)$, write the coordinates of the vector $\overrightarrow{PQ}$.

(4) Dot product of two vectors $\vec{v} = a_1\vec{i} + b_1\vec{j} + c_1\vec{k}$, $\vec{w} = a_2\vec{i} + b_2\vec{j} + c_2\vec{k}$:

    (a) What is the dot product $\vec{v} \cdot \vec{w} = $?

    (b) What does the dot product of two orthogonal vectors equal to?

    (c) Is the dot product a scalar (scalar is another word for "number") or is it a vector?

(5) Vector and scalar projection of a vector $\vec{v}$ onto a vector $\vec{w}$.

Part II. Cross product:

(1) Cross product of two vectors $\vec{v} = a_1\vec{i} + b_1\vec{j} + c_1\vec{k}$, $\vec{w} = a_2\vec{i} + b_2\vec{j} + c_2\vec{k}$, write $\vec{v} \times \vec{w}$ as a determinant and then evaluate the determinant.

(2) Is $\vec{v} \times \vec{w}$ a scalar or is it a vector?

(3) What is the area of the parallelogram determined by two vectors $\vec{v}$ and $\vec{w}$?

(4) Given $\vec{v}$ and $\vec{w}$, how can you find a vector which is orthogonal to both of them?

(5) What is the area of a triangle with vertices P, Q and R in terms of the cross product of two vectors?

Part III. Planes and Lines:

(1) Planes:

    (a) Equation of the plane through the point $P(x_0, y_0, z_0)$ with normal vector $\vec{N} = A\vec{i} + B\vec{j} + C\vec{k}$. *Warning:* you may not be given P or $\vec{N}$ directly, but may have to find them from the data in the problem.

    (b) Equation of a plane through three points P, Q and R which are not aligned (they form a triangle). This is similar to (5) in Part II in the sense that you have to find the cross product of two vectors in order to find a normal vector to the plane.

    (c) Given an equation of a plane, sketch it.

(2) Lines:

    (a) Write the parametric equation of the line through the point $P(x_0, y_0, z_0)$ aligned to the vector $\vec{v} = a\vec{i} + b\vec{j} + c\vec{k}$.

(b) If $a, b, c \neq 0$, write the parametric equation of the line through the point $P(x_0, y_0, z_0)$ aligned to the vector $\vec{v} = a\vec{i} + b\vec{j} + c\vec{k}$.

(c) Find the points of intersection of a line with the coordinate planes.

Part IV. Quadric Surfaces:

(1) Given the equation of a sphere written as $x^2 + y^2 + z^2 + ax + by + cz + d = 0$, complete squares to write it in the form $(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$ and then identify the coordinates of the center and the radius.

(2) Identify the equation and sketch the following:
   (a) Ellipsoid.
   (b) Elliptic paraboloid.
   (c) Parabolic cylinder.

Part V. Distance:

(1) Between two points.
(2) Between a point and a plane.
(3) Between a point and a line.