This is a guide to help you prepare your own summary of convergence methods for series. The best way to use this guide is in study groups trying to explain to each other.

(1) Answer the following questions, they will help you clarify the concepts you will work with:
  (a) What is the meaning of the symbol $\sum_{k=1}^{\infty} a_k$, also called an "infinite series"?

  (b) What is an n-th partial sum for the infinite series $\sum_{k=1}^{\infty} a_k$?

  (c) How is the sequence of partial sums $\{S_n\}$ different from $\{a_k\}$?

  (d) What is meant by "the sum" of an infinite series?

From now on, you may use table 8.1 (Review of convergence Methods) that appears in your textbook on pages 540-541.

(2) Geometric series.

(a) Write the general form of a geometric series: ____________

(b) A geometric series

- Converges if: ____________________________
- Diverges if:____________________________
- If a geometric series converges, its sum is:________

(c) Invent an example of a geometric series, identify a and r in your example and decide whether your series converges or diverges.

(3) Telescoping series:

(a) Write the n-th partial sum of a telescoping series and show how its telescoping.

(b) What examples of telescoping series were studied in class? Try to do those examples on your own and try to understand why the series are telescoping.

(4) p-series

(a) What is a p-series?

(b) A p-series

- Converges if:______________________________
- Diverges if:______________________________

(c) Give an example of a convergent p- series and an example of a divergent one.

(5) Integral Test

(a) For an infinite series $\sum_{k=1}^{\infty} a_k$, what is the associated function $f(x)$? Give some examples.

(b) In order for one to be able to use the integral test for the series $\sum_{k=1}^{\infty} a_k$ one must verify:

  (i) $a_k \geq$ _____

  (ii) a_k is______________________________

(c) Usually, one verifies that the sequence a_k is decreasing by showing either of the following:

  (i) a_{k+1}_____a_k directly

  (ii) The associated function $f(x)$ satisfies that $f'(x)$_____0

  (iii) If one is familiar with the graph of the associated function $f(x)$, one can just draw the graph and it will tell that the associated function, and therefore the sequence, is decreasing.

(d) What improper integral do you need to analyze in the integral test?

(e) What does the integral test say?

(f) Do some examples. **Note:** If the improper integral that you get by replacing k by x is easy to work with (say, there is an obvious substitution or such) then the integral test may work, but you have to verify that you can use it (5b).

(6) Divergence test.

(a) The divergence test says that if the infinite series $\sum_{k=1}^{\infty} a_k$ converges then $\lim_{k\to\infty} a_k =$ _____

(b) If $\lim_{k\to\infty} a_k = 0$, will the series $\sum_{k=1}^{\infty} a_k$ necessarily converge? Give an example where this is not true.

(c) If $\lim_{k\to\infty} a_k = 0$ and the series $\sum_{k=1}^{\infty} a_k$ does converge will the series converge to zero as well? Give an example that shows that this is not true.

(7) Comparison Test

(a) The comparison test says that

(i) If $0 \leq a_k \leq b_k$ and $\sum b_k$ __________________________
then $\sum a_k$ __________________________

(ii) if $0 \leq d_k \leq a_k$ and $\sum d_k$ __________________________
then $\sum a_k$ __________________________

(b) Give examples where the comparison test is useful to decide the convergence or divergence of a series.

(c) The limit comparison test says that if $a_k > 0$, $b_k > 0$ and $\lim_{k\to\infty} \frac{a_k}{bk} = L$, where L satisfies $L \neq$ _____, _____ then the series $\sum a_k$ and $\sum b_k$ both __________________________ or __________________________ at the same time.

(d) Give examples where the limit comparison test is useful to decide the convergence or divergence of a series.

(8) Ratio and root tests.

(a) The ratio test says that if $a_k > 0$ and $\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = L$ then

- (i) If $L <$ _____ the series ____________________
- (ii) If $L >$ _____ or $L =$ _____ the series ____________________
- (iii) If $L =$ _____ the test is ____________________, so you need to try another test.

(b) The root test says that if $a_k \geq 0$ and $\lim_{k\to\infty} \sqrt[k]{a_k} = L$ then

- (i) If $L <$ _____ the series ____________________
- (ii) If $L >$ _____ or $L =$ _____ the series ____________________
- (iii) If $L =$ _____ the test is ____________________, so you need to try another test.