Mathematics 2360
Exam I, Feb 5, 2009
Section _____

Name (please print) ____________________________

Exam points (out of 30): ____________

I. (5) For each of the following linear systems use Gaussian elimination to find all the solutions or to show that the system is inconsistent. If it is inconsistent explain why.

a)
$$\begin{aligned} x_2 + x_3 - 2x_4 - 2x_5 &= -3 \\ x_1 + 2x_2 - x_3 &= 2 \\ 2x_1 + 4x_2 + x_3 - 3x_4 - 3x_5 &= -2 \\ x_1 - 4x_2 - 7x_3 - x_4 - x_5 &= -19 \end{aligned}$$

b)
$$\begin{aligned} x_1 - x_2 + 2x_3 &= 4 \\ x_1 \qquad + x_3 &= 6 \\ 2x_1 - 3x_2 + 5x_3 &= 4 \\ 3x_1 + 2x_2 - x_3 &= 1 \end{aligned}$$

II. Answer the following questions:
(6)

a) What is an elementary matrix of order n?

b) What does it mean that a matrix A has an inverse?

c) What does it mean that two $n \times n$ matrices A and B are row equivalent?

d) If a matrix A is row equivalent to a matrix B and B is row equivalent to a matrix C, what can be said about A and C? Justify your answer with a rigorous proof.

e) Is a matrix A row equivalent to itself? Justify your answer with a rigurous proof.

f) If A is an $n \times n$ matrix and α is a scalar, then $\det(\alpha A) = \alpha^n \det(A)$

III. (5) Given $A = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 3 & 5 \\ 2 & 4 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 2 & 1 \\ 5 & 4 \end{pmatrix}$, compute the following matrices if possible. When not possible, indicate so and justify your answer.

First, write $A^T = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$, $B^T = \begin{pmatrix} & & \\ & & \end{pmatrix}$ and $I = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$, where I denotes the 3×3-identity matrix.

a)$A + I$

b)$B^T A$

c)$A^T B^T$

d)B^2

e)BB^T

IV. For the matrix
(7)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 6 \\ 1 & 3 & 4 \end{bmatrix}$$

a) Find elementary matrices E_1, E_2 and E_3 such that $E_3 E_2 E_1 A = U$ is an upper triangular matrix.

b) Find the inverses of the matrices E_1, E_2 and E_3.

c) Find a lower triangular matrix L such that $A = LU$, where U is the matrix found in part a).

V. Find the inverse of the matrix $A = \begin{pmatrix} 1 & 3 & -12 \\ -2 & -1 & 6 \\ -1 & 0 & 1 \end{pmatrix}$
(5)

VI. Use Gaussian elimination to find the $\det(A)$ where
(5)

$$A = \begin{pmatrix} 2 & 0 & 0 & 1 \\ 0 & 1 & 3 & -3 \\ -2 & -3 & -5 & 2 \\ 4 & -4 & 4 & -6 \end{pmatrix}$$