Mathematics 2360 Name (please print) ______________

Quiz 9-a

Apr 15, 2010

I. (3) Find a basis for the nullspace of the matrix $A = \begin{pmatrix} 1 & 1 & -1 & 2 \\ 2 & 2 & -3 & 1 \\ -1 & -1 & 0 & -5 \end{pmatrix}$.

II. (2) Find a basis for the span of the following set of vectors: $\{(1,-3,1,1)^T, (2,1,-1,2)^T, (1,4,-2,1)^T, (5,-8,2,5)^T\}$

III. (3) Decide whether the vectors $\{x^2-2x+3, 2x^2+x+8, x^2+8x+7, x^2-15x-1\}$ are linearly dependent in P_3. If so, find a nontrivial linear combination that gives the zero vector. If they are independent justify your answer.

I. Nullspace: All $\vec{x}$ for which $A\vec{x}=\vec{0}$ $N(A)$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 2 & 0 \\ 2 & 2 & -3 & 1 & 0 \\ -1 & -1 & 0 & -5 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{cccc|c} 1 & 1 & 0 & 5 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

x_2, x_4 free variables

$x_2 = s$ $x_1 = -s-5t$

$x_4 = t$ $x_3 = -3t$

$$(-s-5t, s, -3t, t)^T = s(-1,1,0,0)^T + t(-5,0,-3,1)^T$$

$\therefore$ $\{(-1,1,0,0)^T, (-5,0,-3,1)^T\}$ is a basis for $N(A)$

II. $$\begin{bmatrix} 1 & 2 & 1 & 5 \\ -3 & 1 & 4 & -8 \\ 1 & -1 & -2 & 2 \\ 1 & 2 & 1 & 5 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & -1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

So $\{(1,-3,1,1)^T, (2,1,-1,2)^T\}$ is a basis for the set of vectors.

III. This is a set of 4 vectors in P_3, so they are linearly dependent.

$$(3)(x^2-2x+3)+(-2)(2x^2+x+8)+(1)(x^2+8x+7)+(0)(x^2-15x-1)=0$$