

- I. Given $A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 4 & 3 \\ 1 & 2 & 2 \end{pmatrix}$ determine the (3,1) entry of A^{-1} by computing a quotient of two determinants.
(2)

Clearly indicate the two determinants involved.

$$A^{-1} = \frac{1}{\det(A)} \text{Adj}(A)$$

$$\text{Adj}(A) = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ \boxed{A_{13}} & A_{23} & A_{33} \end{bmatrix}$$

$$A_{ij} = (-1)^{i+j} \det(M_{ij})$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 0 & 4 \\ 1 & 2 \end{vmatrix} = -4$$

$$\frac{A_{13}}{\det(A)} = \frac{-4}{4} = \boxed{-1}$$

II. Let A be the matrix in problem I. Compute the first column of A^{-1} by using Cramer's rule to solve

(3)

$$Ax = e_1, \text{ where } e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}.$$

$$A_1 = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 4 & 3 \\ 0 & 2 & 2 \end{bmatrix}$$

$$\det(A_1) = 2$$

$$A_2 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 3 \\ 1 & 0 & 2 \end{bmatrix}$$

$$\det(A_2) = -(-3) = 3$$

$$A_3 = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 4 & 0 \\ 1 & 2 & 0 \end{bmatrix}$$

$$\det(A_3) = -4$$

$$\text{First column of } A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -4 \end{bmatrix}$$