Mathematics 2360 Name (please print) ______________________

Quiz 6-a

Mar 04, 2010

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**I.** (2) Given $A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 4 & 3 \\ 1 & 2 & 2 \end{pmatrix}$ determine the (1,2) entry of $A^{-1}$ by computing a quotient of two determinants. Clearly indicate the two determinants involved.

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$$A^{-1} = \frac{1}{\det(A)} \operatorname{Adj}(A)$$

$$\operatorname{Adj}(A) = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$A_{ij} = (-1)^{i+j} \det(M_{ij})$$

$$A_{12} = (-1)^{2+1} \begin{vmatrix} 2 & 1 \\ 2 & 2 \end{vmatrix} = -2$$

$$\frac{A_{12}}{\det(A)} = \frac{-2}{4} = -\frac{1}{2}$$

**II.** (3) Let $A$ bet the matrix in problem I. Compute the ~~first~~ 2nd column of $A^{-1}$ by using Cramer's rule to solve

$$A\mathbf{x} = \mathbf{e}_2, \text{ where } \mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}.$$

$$A_1 = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 4 & 3 \\ 0 & 2 & 2 \end{bmatrix} \qquad \det(A_1) = 2$$

$$A_2 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 3 \\ 1 & 0 & 2 \end{bmatrix} \qquad \det(A_2) = -(-3) = 3$$

$$A_3 = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 4 & 0 \\ 1 & 2 & 0 \end{bmatrix} \qquad \det(A_3) = -4$$

First column of $A^{-1} = \dfrac{1}{\det(A)} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -4 \end{bmatrix}$