Mathematics 2360
Quiz 3
Feb 04, 2010

Name (please print) key

I. (3) Suppose that

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad \text{and} \quad A^{-1} = \frac{1}{d}\begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}$$

are inverses of each other. Find d.

$$AA^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} \frac{a_{22}}{d} & -\frac{a_{12}}{d} \\ -\frac{a_{21}}{d} & \frac{a_{11}}{d} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{a_{11}a_{22}-a_{12}a_{21}}{d} & 0 \\ 0 & \frac{a_{11}a_{22}-a_{12}a_{21}}{d} \end{bmatrix}$$

$$\frac{a_{11}a_{22}-a_{12}a_{21}}{d} = 1$$

So $d = a_{11}a_{22} - a_{12}a_{21}$

II. (1) Use the form of A^{-1} in problem I to find the inverse of $A = \begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$, i.e. $A^{-1} = \begin{pmatrix} 2 & -3 \\ -3 & 5 \end{pmatrix}$

$d = \underline{1}$

III. (1) Let A and B be $n \times n$ matrices. Show that if

$$AB = A \quad \text{and} \quad B \neq I$$

then A must be singular.

If A is nonsingular we can multiply both sides by its inverse:

$$A^{-1}(AB) = A^{-1}A$$

$$(A^{-1}A)B = I$$

Thus $B = I$.