Mathematics 2360
Quiz 2
Jan 28, 2010

Name (please print) Solutions

I. (Bonus!) Let A be an $m \times n$ matrix. Does the product AA^T exist? Explain.
(2) The product AA^T exists b/c the number of columns of A equals the number of rows of A^T.

II. Let $A\mathbf{x} = \mathbf{b}$ be a linear system whose augmented matrix $(A|\mathbf{b})$ has reduced row echelon form
(5)

$$\left(\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 4 \\ 0 & 0 & 1 & 1 & 0 & -6 \\ 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

x_2 and x_4 are free variables, so we let $x_2 = s$, $x_4 = t$.

$x_1 = 4 - s$
$x_2 = s$
$x_3 = -6 - t$
$x_4 = t$
$x_5 = 3$

a)Find all solutions to the system. Solutions: $(4-s, s, -6-t, t, 3)$ where s and t are any real numbers.

b)If

$$\mathbf{a_1} = \begin{pmatrix} 1 \\ -1 \\ -2 \\ 0 \\ 1 \end{pmatrix}, \mathbf{a_3} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 2 \end{pmatrix}, \mathbf{a_5} = \begin{pmatrix} 1 \\ 1 \\ 3 \\ 3 \\ 4 \end{pmatrix},$$

find $\mathbf{b}$.

Since $\underline{b} = x_1\underline{a}_1 + x_2\underline{a}_2 + x_3\underline{a}_3 + x_4\underline{a}_4 + x_5\underline{a}_5$ and $\underline{a}_2$ and $\underline{a}_4$ correspond to free variables, we can let $s = 0$, $t = 0$ to get $x_1 = 4$, $x_3 = -6$, $x_5 = 3$

therefore $\underline{b} = 4\underline{a}_1 - 6\underline{a}_3 + 3\underline{a}_5 = \begin{pmatrix} 1 \\ -1 \\ 1 \\ 3 \\ 4 \end{pmatrix}$