Corrections and Explanations for Luan Hoang's publications January 28, 2025

A. The Navier–Stokes Equations

- 1. Paper [6], formula (1.13): remove the last "= 0" (for B_j).
- 2. Paper [13], sentence after (4.58): insert " u_{N+1} " after "since", and add the terms "multiplied by $e^{-\mu_{N+1}t}$ " after " G_{μ,σ_0} -valued polynomial".

Explanations: Summing up (4.58) is permitted by simply working with the coefficients of the polynomials.

- 3. Paper [14]:
 - (a) Page 113, line 3: replace "linear" with "bilinear"
 - (b) After (3.15), in the formula of ξ_1 : remove $e^{\mu_1 \tau}$ in the integral.
 - (c) Before (3.33), in the formula of ξ_{N+1} : remove $e^{\mu_{N+1}\tau}$ in the integral.
- 4. Paper [7]:
 - (a) The end of (2.11): \mathbb{R}^n should be \mathbb{R}^3 .
 - (b) At the bottom of page 990, repaire $(e^{-(\mu_N + \varepsilon_N/2)t}))$ with $\mathcal{O}(e^{-(\mu_N + \varepsilon_N/2)t})$

5. Paper [8]:

- (a) Page 5, after (2.4): replace "eigenvector" with "eigenvalue".
- (b) Page 20, line 4: change $\mathcal{E}_{\mathbb{K}}(m,k,-\mu)$ to $\mathcal{E}_{\mathbb{C}}(m,k,-\mu)$
- (c) Page 21, line 5: change $\mathcal{E}_{\mathbb{C}}(m, N, -\mu)$ to $\mathcal{E}_{\mathbb{C}}(m, k, -\mu)$
- (d) Page 21, line 6: change $\mathcal{E}_{\mathbb{K}}(m, N, -\mu)$ to $\mathcal{E}_{\mathbb{C}}(m, k, -\mu)$
- (e) Page 25, after (5.25): $(\mathcal{Z}_{A_{\mathbb{C}}}p)... \in G_{\alpha+1,\sigma}$ should be $(\mathcal{Z}_{A_{\mathbb{C}}}p)... \in G_{\alpha+1,\sigma,\mathbb{C}}$
- (f) Page 30, 2nd line after (6.30): replace T_{*} > E_k(0) with u(t) is a Leray-Hopf weak solution on [T_{*},∞)
- (g) Page 31, (6.33): replace C with C^2
- (h) Page 35, lines -1 and -2: $\sum_{k=1}^{N}$ should be $\sum_{\lambda=1}^{N}$

B. Porous Media Equations

- 1. Paper [10]:
 - (a) Page 2, line 2: "may" should be "many"
- 2. Paper [1], formula (37) and line 1 of the next page: replace $(K(\xi)\xi^n)$ with derivative $(K(\xi)\xi^n)'$
- 3. Paper [9], Lemma 2.3, inequality (2.24) and first inequality of part (ii): replace " $\geq a$ " with " $\geq (1-a)$ "
- 4. Paper [12]:
 - (a) Page 279, line 3: insert "+2s" in front of the integral.
 - (b) Page 330, the line above (A.10): \tilde{A} should be $(A_1 D^{\mu_1} + A_2 D^{\mu_2})/D^{\mu_1}$.
- 5. Paper [11]:
 - (a) Page 17, inequalities (128) and (132): "− inf |w(x,0)|" should be "inf(−|w(x,0)|)" and, hence, "− sup |w(x,0)|".
- 6. Paper [5]:
 - (a) Lemma A.1: Because $\prod \gamma_j$ is convergent and by the Cauchy criterion, the sequence $(G_j)_{j=1}^{\infty}$ is bounded. Hence number G is finite.
- 7. Paper [4]:
 - (a) Inequalities (3.11) on page 3616 and (3.44) on page 3631, insert constant $\alpha(\alpha \lambda)d_3/(8\lambda)$ before the integral $\int_U |\nabla u|^{2-a}u^{\alpha-\lambda-1}dx$ on the left.
 - (b) Page 3629, after (3.28), reference (3.25) should be (3.28).
 - (c) Line 3 from the bottom of page 3632, line 3 of page 3633, and line 2 of page 3635: The constant C should be \bar{C} .
 - (d) Page 3633, inequality (4.6): the last two terms should be multiplied by 2.
 - (e) In inequalities (4.8) on page 3633 and (4.9) on page 3634, the integrand of the integrals $\iint_{Q_T} u^{\alpha-\lambda-1} |\nabla u|^{2-a} dx dt \text{ should be multiplied by } \xi^2.$
 - (f) Page 3634, after (4.11): value of ε should be $(\alpha \lambda)d_3/32$.

C. Ordinary Differential Equations

- 1. Paper [2]:
 - (a) Page 1195: In (3.5), $\varepsilon_0 = C_0/2$.
- 2. Paper [3]:
 - (a) Page 17: 2nd line after (4.25), $\sum_{j=1}^{m} \mu_{k_j}$ should be $\sum_{j=1}^{m} \widetilde{\mu}_{k_j}$
 - (b) Page 17: after (4.26), "condition (4.27)" should be "condition (4.26)"
 - (c) Page 20, last line, and page 21, lines 3 and 6: " z_N " should be " z_m ".

References

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