Texas Tech University. Department of Mathematics and Statistics.
Colloquium.

On nonlinear parabolic equations of the p-Laplace type

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ABSTRACT. The subject of my talk is the regularity theory for nonlinear parabolic equations of the p-Laplace type. The prototype p-Laplace equation $u_t = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ first appeared in modeling of the heat transfer in the presence of very high gradients of temperature and later found application in other areas of mathematical physics including mathematical hydrodynamics, biology, theory of flows in porous media and so on. Equations with the principal part of this type, while inheriting many generic traits from the heat equation, exhibit many new unexpected features. For instance, for $p > 2$ the velocity of propagation of disturbances is finite, while for $p < 2$ a solution can become “extinct” in finite time.

First, I discuss regularity properties of solutions in the spirit of the theory developed by E. De Giorgi, J. Moser, O. Ladyzhenskaya and N. Uraltseva, E. DiBenedetto and many other distinguished authors. The focus of my research in this direction is studying weighted equations, which can possess singularity or degenerate with respect to the spatial variable. This generates an interplay between the weight and the core singularity/degeneracy with respect to the gradient in the principal part.

Second, I present my research on the global solvability of the Cauchy problem for equations of the p-Laplace type in the whole space. This is achieved in terms of weighted energy spaces. In close connection with the question of existence of a global solution stands the question of describing its asymptotic behavior. In this respect I give a criterion for the uniform stabilization of solutions.

I conclude the talk by presenting my results on the question of density of smooth functions in weighted Sobolev spaces with variable exponent. These spaces are induced by the energy $\int_\Omega |\nabla u|^{p(x)}\rho(x)\,\mathrm{d}x$. Formally, this topic belongs rather to the area of real analysis. Nevertheless, it is deeply connected to nonlinear PDEs. Spaces of this type arise naturally when one studies equations involving the weighted $p(x)$-Laplacian operator. I give a sufficient condition on the weight which guarantees that smooth functions are dense in the Sobolev space. This condition is stated in terms of the asymptotic behavior of large positive and negative powers of the weight.