Solution

MATH2450. SECTION 010. FALL 2012.

MIDTERM EXAMINATION 2-A

Name (in CAPITALS): ....

Signature: .... Date: ....

READ AND FOLLOW THESE INSTRUCTIONS

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INSTRUCTIONS FOR MULTIPLE CHOICE PROBLEMS (Questions 1 – 4):
There are 4 multiple choice problems, worth 15 points each.
You must indicate your answers clearly.
When you have decided on a correct answer to a given question, circle the answer. Each question has a correct answer. If you give two different answers, the question will be marked wrong. There is no penalty for guessing.

INSTRUCTIONS FOR THE WORK OUT PROBLEMS (Questions 5 – 6):
There are 2 work-out problems, worth 20 points each.
SHOW ALL WORK. Unsupported answers will receive little credit.

AFTER YOU FINISH BOTH PARTS OF THE EXAM; turn in the whole booklet.

PART I. Multiple Choice Problems. (Questions 1 – 4)

1. (15 points) Find the limit

$$\lim_{(x,y)\to(0,0)} \frac{2x^3y}{x^4+3y^4}.$$

(a) 1/2

(b) 0

(c) 2/3

(d) ∞

(e) Does not exist

$y = ax$

$$\frac{2x^3ax}{x^4+3a^4x^4} = \frac{2a}{1+3a^4}$$

2. (15 points) Let

$$f(x,y) = 2x^2y \text{ and } \mathbf{u} = \frac{1}{\sqrt{5}}\langle -2, -1\rangle.$$ $|u| = 1$

Find the directional derivative $D_{\mathbf{u}}f(1,1)$.

(a) $\langle 4, 2\rangle$

(b) $-2\sqrt{5}$

(c) -10

(d) $-2/\sqrt{5}$

(e) $-3/\sqrt{5}$

$f_x = 4xy \qquad f_x(1,1) = 4$

$f_y = 2x^2 \qquad f_y(1,1) = 2$

$$D_u f(1,1) = \nabla f(1,1) \cdot u$$
$$= \frac{4(-2) + 2(-1)}{\sqrt{5}}$$
$$= \frac{-10}{\sqrt{5}} = -2\sqrt{5}$$

3. (15 points) Let $z = z(x, y)$ be a function of x and y and be defined implicitly by the equation

$$x^2 + y^3 + xyz^2 = 1.$$

Find the partial derivative $\dfrac{\partial z}{\partial x}$.

(a) $-\dfrac{2x + yz^2}{2xyz}$

(b) $\dfrac{1 - x^2 - y^3}{2xyz}$

(c) $-\dfrac{1}{yz}$

(d) $\dfrac{1 - 2x - yz^2}{2xyz}$

(e) $-\dfrac{3y^2 + xz^2}{2xyz}$

Take $\dfrac{\partial}{\partial x}$ of the equation

$$2x + 0 + yz^2 + \underline{xy \cdot 2z}\, z_x = 0$$

$$z_x = \frac{-2x - yz^2}{2xyz}$$

4. (15 points) Let $z = \ln(3x - y^2)$, where $x = u^2 v$ and $y = 2u - v^2$. Use the chain rule to find $\dfrac{\partial z}{\partial u}$.

(a) $\dfrac{6uv}{3x - y^2}$

(b) $\dfrac{2uv + 2}{3x - y^2}$

(c) $\dfrac{2 - 2y}{3x - y^2}$

(d) $\dfrac{6uv - 4y}{3x - y^2}$

(e) $\dfrac{2uv}{3x - y^2}$

$$\frac{\partial z}{\partial x} = \frac{3}{3x-y^2} \qquad \frac{\partial z}{\partial y} = \frac{-2y}{3x-y^2}$$

$$\frac{\partial x}{\partial u} = 2uv \qquad \frac{\partial y}{\partial u} = 2$$

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial u}$$

$$= \frac{3(2uv)}{3x-y^2} - \frac{2y}{3x-y^2}(2)$$

$$= \frac{6uv-4y}{3x-y^2}$$

PART II. Work Out Problems. (Questions 5 – 6)

5. (20 points) Let

$$f(x,y) = x^2 - 4xy + \frac{4}{3}y^3 - 1.$$

d

Find all critical points and use the second partials test to classify each point as a relative maixum, a relative minimum or a sadle point.

$f_x = 2x - 4y$ $\quad$ $f_y = -4x + 4y^2$

$f_x = 0 \Leftrightarrow x = 2y$ $\quad$ $f_y = 0$ $\quad$ $-x + y^2 = 0$

$\hookrightarrow$ $\quad$ $-2y + y^2 = 0$

$y = 0, 2$

$\downarrow$

$x = 0, 4$

Critical points:

$\Rightarrow$ $(0,0)$ and $(4,2)$

$f_{xx} = 2$ $\quad$ $f_{yy} = 8y$

$f_{xy} = -4$

$(0,0)$

$$D = \begin{vmatrix} 2 & -4 \\ -4 & 0 \end{vmatrix} = -16 < 0$$

$\Downarrow$

Saddle point

$(4,2)$

$$D = \begin{vmatrix} 2 & -4 \\ -4 & 16 \end{vmatrix} = 16 > 0$$

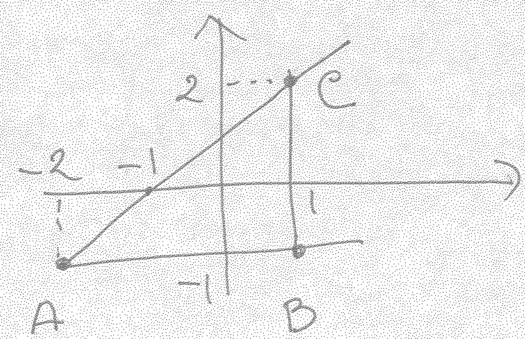
$f_{xx} = 2 > 0$

$\Rightarrow$ relative min

be

6. (20 points) Let $f(x, y) = x^2 - y^2$ and the domain D is the triangle ABC, where the vertices are $A(-2, -1)$, $B(1, -1)$ and $C(1, 2)$, that is, the domain D is the region bounded by $y = -1$, $x = 1$, and $y = x + 1$.

Find the absolute maximum and absolute minimum of $f(x, y)$ over the domain D.



$f_x = 2x$, $f_y = -2y$

Critical points: $(0,0)$

On (AB) $y = -1$,

$f = x^2 - 1 = F_1(x)$, $-2 \le x \le 1$

$F_1'(x) = 2x$ $\quad F_1'(x) = 0 \Leftrightarrow x = 0$

$\longrightarrow$ point $(0, -1)$

End points: A, B

On (BC) $x = 1$

$f = 1 - y^2 = F_2(y)$, $-1 \le y \le 2$

$F_2'(y) = -2y$, $F_2'(y) = 0 \Leftrightarrow y = 0$

$\Rightarrow$ point $(1, 0)$

End points: B, C

On (AC): $y = x + 1$

$f = x^2 - (x+1)^2$

$= -2x - 1 = F_3(x)$

$-2 \le x \le 1$

$F_3'(x) = -2 \ne 0$

End points A, C

Table

(x, y)	$f(x,y)$
$(0, 0)$	0
$(0, -1)$	-1
$(1, 0)$	1
$A(-2, -1)$	$3 \to$ largest
$B(1, -1)$	0
$C(1, 2)$	$-3 \to$ smallest

Hence

Absolute max $= 3$ (at A)

Absolute min $= -3$ (at C)

Solution

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MIDTERM EXAMINATION 2-B

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PART I. Multiple Choice Problems. (Questions 1 – 4)

1. (15 points) Find the limit

$$\lim_{(x,y)\to(0,0)} \frac{3x^2y^2}{4x^4+y^4}.$$

(a) 0

(b) 3/5

(c) ∞

(d) 3/4

(e) Does not exist

$y = ax$

$$\frac{3x^2a^2x^2}{4x^4+a^4x^4} = \frac{3a^2}{4+a^4}$$

2. (15 points) Let

$$f(x, y) = 2xy^3 \text{ and } \mathbf{u} = \frac{1}{\sqrt{5}}\langle -1, 2\rangle.$$

$|u| = 1$

Find the directional derivative $D_{\mathbf{u}}f(1, 1)$.

(a) 10

(b) $\langle 2, 6\rangle$

(c) $2\sqrt{5}$

(d) $-2/\sqrt{5}$

(e) $4/\sqrt{5}$

$f_x = 2y^3 \quad f_x(1,1) = 2$

$f_y = 6xy^2 \quad f_y(1,1) = 6$

$$D_u f(1,1) = \nabla f(1,1) \cdot u$$

$$= \frac{2(-1) + 6(2)}{\sqrt{5}}$$

$$= \frac{10}{\sqrt{5}} = 2\sqrt{5}$$

3. (15 points) Let $z = z(x, y)$ be a function of x and y and be defined implicitly by the equation

$$x^2 + y^3 + xyz^2 = 2.$$

Find the partial derivative $\dfrac{\partial z}{\partial y}$.

(a) $-\dfrac{2x + yz^2}{2xyz}$

(b) $\dfrac{2 - x^2 - y^3}{2xyz}$

(c) $\dfrac{2 - 3y^2 - xz^2}{2xyz}$

(d) $-\dfrac{3y^2 + xz^2}{2xyz}$

(e) $-\dfrac{3y}{2xz}$

Take $\dfrac{\partial}{\partial y}$ of the equation

$$0 + 3y^2 + xz^2 + \underline{xy2z}\, z_y = 0$$

$$z_y = \frac{-3y^2 - xz^2}{2xyz}$$

4. (15 points) Let $z = \ln(y^2 - 4x)$, where $x = u^2 v$ and $y = v^2 - 3u$. Use the chain rule to find $\frac{\partial z}{\partial v}$.

(a) $\frac{-4u^2}{y^2 - 4x}$

(b) $\frac{-4u^2 + 4yv}{y^2 - 4x}$

(c) $\frac{u^2 + 2v}{y^2 - 4x}$

(d) $\frac{u^2}{y^2 - 4x}$

(e) $\frac{2y - 4}{y^2 - 4x}$

$$\frac{\partial z}{\partial x} = \frac{-4}{y^2 - 4x}$$

$$\frac{\partial z}{\partial y} = \frac{2y}{y^2 - 4x}$$

$$\frac{\partial x}{\partial v} = u^2 \qquad \frac{\partial y}{\partial v} = 2v$$

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}$$

$$= \frac{-4}{y^2-4x}(u^2) + \left(\frac{2y}{y^2-4x}\right)\cdot 2v$$

$$= \frac{-4u^2 + 4yv}{y^2 - 4x}$$

PART II. Work Out Problems. (Questions 5 – 6)

5. (20 points) Let

$$f(x,y) = y^2 - 4xy + \frac{4}{3}x^3 + 2.$$

Find all critical points and use the second partials test to classify each point as a relative maixum, a relative minimum or a saddle point.

$f_x = -4y + 4x^2 \qquad f_y = 2y - 4x$

Critical points

$f_y = 0 \iff y = 2x$

$f_x = 0 \iff -y + x^2 = 0$

$-2x + x^2 = 0$

$x = 0, 2$

$\rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases} \qquad \begin{cases} x = 2 \\ y = 4 \end{cases} \qquad f_{xx} = 8x, \quad f_{yy} = 2, \quad f_{xy} = -4$

$(0,0)$

$D = \begin{vmatrix} 0 & -4 \\ -4 & 2 \end{vmatrix} = -16 < 0$

$\Downarrow$

Saddle point

$(2,4)$

$D = \begin{vmatrix} 16 & -4 \\ -4 & 2 \end{vmatrix} = 16 > 0$

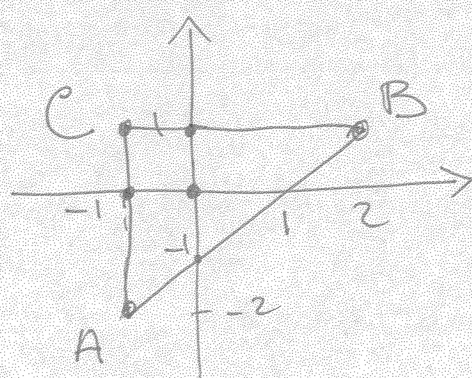
$f_{xx} = 16 > 0$

$\Downarrow$

relative min

6. (20 points) Let $f(x,y) = y^2 - x^2$ and the domain D be the triangle ABC, where the vertices are $A(-1,-2)$, $B(2,1)$ and $C(-1,1)$, that is, the domain D is the region bounded by $x=-1$, $y=1$, and $y=x-1$.

Find the absolute maximum and absolute minimum of $f(x,y)$ over the domain D.



$f_x = -2x$, $f_y = 2y$

Critical point: $(0,0)$

On (AC) $x=-1$

$f = y^2 - 1 = F_1(y)$ $-2 \le y \le 1$

$F_1'(y) = 2y$ $F_1'(y) = 0 \Leftrightarrow y = 0$

$\Rightarrow$ point $(-1,0)$

End points: A, C

On (CB) $y=1$

$f = 1 - x^2 = F_2(x)$

$-1 \le x \le 2$

$F_2'(x) = -2x$

$F_2'(x) = 0 \Leftrightarrow x = 0$

$\Rightarrow$ point $(0,1)$

End points: C, B

On (AB)

$f = (x-1)^2 - x^2 = -2x + 1 = F_3(x)$

$-2 = F_3'(x) \neq 0$

End points A, B

Table

(x,y)	$f(x,y)$	
$(0,0)$	0	
$(-1,0)$	-1	
$(0,1)$	1	
$A(-1,-2)$	3	$\rightarrow$ largest
$B(2,1)$	-3	$\rightarrow$ smallest
$C(-1,1)$	0	

Hence

Absolute max $= 3$ (at A)

Absolut min $= -3$ (at B)