SOLUTIONS

MATH2450. SECTION 010. FALL 2012.

MIDTERM EXAMINATION 1-A

Name (in CAPITALS): ....

Signature: ....Date: ....

READ AND FOLLOW THESE INSTRUCTIONS

This booklet contains 7 pages, including this cover page. Check to see if any are missing. PRINT all the requested information above, and sign your name. Put your initials on the top of every page, in case the pages become separated.

This is a closed-book examination. No books, no notes, no formula sheets, no calculators.

Do your work in the blank spaces and back of pages of this booklet.

INSTRUCTIONS FOR MULTIPLE CHOICE PROBLEMS (Questions 1 – 4):

There are 4 multiple choice problems, worth 15 points each.

You must indicate your answers clearly.

When you have decided on a correct answer to a given question, circle the answer. Each question has a correct answer. If you give two different answers, the question will be marked wrong. There is no penalty for guessing.

INSTRUCTIONS FOR THE WORK OUT PROBLEMS (Questions 5 – 6):

There are 2 work-out problems, worth 20 points each.

SHOW ALL WORK. Unsupported answers will receive little credit.

AFTER YOU FINISH BOTH PARTS OF THE EXAM; turn in the whole booklet.

PART I. Multiple Choice Problems. (Questions 1 – 4)

1. (15 points) Find the equation for the straight line through two points $P(2, 2, 1)$ and $Q(0, 3, -1)$.

(a) $\frac{x+2}{-2} = \frac{y+2}{1} = \frac{z+1}{-2}$

(b) $-2x + y - 2z + 5 = 0$

(c) $x = 2 + 2t,\ y = 2 + 5t,\ z = 1$

(d) $x = 2t,\ y = 3 + 2t,\ z = -1 + t$

(e) $x = 2 - 2t,\ y = 2 + t,\ z = 1 - 2t$ *

$v = PQ = \langle 0-2, 3-2, -1-1 \rangle$

$= \langle -2, 1, -2 \rangle$

Using $P(2,2,1)$:

$$\begin{cases} x = 2 - 2t \\ y = 2 + t \\ z = 1 - 2t \end{cases}$$

2. (15 points) Find the equation of the plane through the point $P(-1,1,1)$ and parallel to two vectors $\mathbf{v} = \langle 1,2,2\rangle$ and $\mathbf{w} = \langle -2,-1,1\rangle$.

(a) $4(x+1) - 5(y+1) + 3(z-1) = 0$

(b) $4x - 5y + 3z + 6 = 0$ *

(c) $3x + 4y + 3z - 4 = 0$

(d) $4x + 5y + 3z + 6 = 0$

(e) $-5y + 3z + 2 = 0$

$$N = V \times W = \begin{vmatrix} i & j & k \\ 1 & 2 & 2 \\ -2 & -1 & 1 \end{vmatrix}$$

$$= (2-(-2))\, i - (1-(-4))\, j + (-1-(-4))\, k$$

$$= \langle 4, -5, 3 \rangle$$

Using point-normal form:

$$4(x+1) - 5(y-1) + 3(z-1) = 0$$

$$4x - 5y + 3z + \underbrace{4+5-3}_{} = 0$$

$$4x - 5y + 3z + 6 = 0$$

3. (15 points) Find the length of the curve $\mathbf{R}(t) = \langle t^2\,,\,-4t^2\,,\,2t^2 \rangle$ over the interval $[0, 2]$.

(a) 10

(b) $4\sqrt{21}$ *

(c) $8\sqrt{21}/3$

(d) 28

(e) $8/3$

$$R(t) = t^2 \langle 1, -4, 2 \rangle$$

$$R'(t) = 2t \langle 1, -4, 2 \rangle$$

$t \in [0,2]$: $\|R'(t)\| = 2t\sqrt{1+16+4}$

$$= 2t\sqrt{21}$$

$$s = \int_0^2 \|R'(t)\|\,dt = \sqrt{21}\int_0^2 2t\,dt$$

$$= \sqrt{21}\; t^2\Big|_0^2$$

$$= 4\sqrt{21}$$

4. (15 points) Let $\mathbf{v}(t) = \langle e^{2t}, \sin t, t^3 \rangle$ be the velocity vector and $\mathbf{R}(0) = \langle -1, 2, 2 \rangle$ be the initial position vector. Find the position vector $\mathbf{R}(t)$.

(a) $\left\langle 2e^{2t} - 1, \cos t + 2, 3t^2 + 2 \right\rangle$

(b) $\left\langle \frac{e^{2t} - 2}{2}, -\cos t + 1, \frac{t^4}{4} \right\rangle$

(c) $\left\langle e^{2t} - 1, \cos t + 2, t^3 + 2 \right\rangle$

(d) $\left\langle \frac{e^{2t} - 3}{2}, -\cos t + 3, \frac{t^4}{4} + 2 \right\rangle$ *

(e) $\left\langle e^{2t} - 2, \cos t + 1, t^3 + 2 \right\rangle$

$$R(t) = R(0) + \int_0^t V(\tau)\,d\tau$$

$$= \left\langle -1 + \int_0^t e^{\cancel{2t}}\,d\tau,\ 2 + \int_0^t \sin\tau\,d\tau,\ 2 + \int_0^t \tau^3\,d\tau \right\rangle$$

$$= \left\langle -1 + \frac{1}{2}(e^{2t} - 1),\ 2 - \cos\tau\Big|_0^t,\ 2 + \frac{t^4}{4} \right\rangle$$

$$= \left\langle \frac{1}{2}e^{2t} - \frac{3}{2},\ \underbrace{2 - \cos t + 1}_{-\cos t + 3},\ 2 + \frac{t^4}{4} \right\rangle$$

PART II. Work Out Problems. (Questions 5 – 6)

5. (20 points) Find the limit, if it exists

$$\lim_{x\to 0}\left\langle \frac{1+x^2}{2-x}, \frac{\ln(2x+1)}{x}, \frac{3x}{\sin(2x)} \right\rangle$$

L'Hospital

$$\left\langle \frac{1}{2}, 2, \frac{3}{2} \right\rangle.$$

6. (20 points) Find the curvature $\kappa(t)$ of the following curve

$$R(t) = \Big\langle \cos(2t) - 1,\, 3t\,,\, \sin(2t) - 1 \Big\rangle$$

$$R' = \langle -2\sin 2t, 3, 2\cos 2t \rangle$$

$$R'' = \langle -4\cos 2t, 0, -4\sin 2t \rangle$$

$$R' \times R'' = \langle -12\sin 2t, -8, 12\cos 2t \rangle$$

$$= 4\langle -3\sin 2t, -2, 3\cos 2t \rangle$$

$$\|R'\| = \sqrt{4\sin^2 2t + 4\cos^2 2t + 9} = \sqrt{9+4} = \sqrt{13}$$

$$\|R' \times R''\| = 4\sqrt{9\sin^2 2t + 9\cos^2 2t + 4}$$

$$= 4\sqrt{13}$$

$$\kappa = \frac{\|R' \times R''\|}{\|R'\|^3} = \frac{4\cancel{\sqrt{13}}}{13\cancel{\sqrt{13}}} = \frac{4}{13}.$$

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MIDTERM EXAMINATION 1-B

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AFTER YOU FINISH BOTH PARTS OF THE EXAM; turn in the whole booklet.

PART I. Multiple Choice Problems. (Questions 1 - 4)

1. (15 points) Find the equation for the straight line through two points $P(1,2,-1)$ and $Q(3,1,0)$.

(a) $\frac{x+1}{2} = \frac{y+2}{-1} = \frac{z-1}{1}$

(b) $2x - y + z + 1 = 0$

(c) $x = 1 + 2t,\ y = 2 - t,\ z = -1 + t$ *

(d) $x = 1 + 4t,\ y = 2 + 3t,\ z = -1 - t$

(e) $x = 3 + t,\ y = 1 + 2t,\ z = -t$

$$V = PQ = \langle 3-1, 1-2, 0-(-1) \rangle$$
$$= \langle 2, -1, 1 \rangle$$

Using $P(1,2,-1)$:

$$\begin{cases} x = 1 + 2t \\ y = 2 - t \\ z = -1 + t \end{cases}$$

2. (15 points) Find the equation of the plane through the point $P(1,1,-1)$ and parallel to two vectors $\mathbf{v} = \langle 1,2,-1\rangle$ and $\mathbf{w} = \langle 2,1,2\rangle$.

(a) $5x - 4y - 3z - 4 = 0$ *

(b) $5(x-1) - 4(y+1) - 3(z+1) = 0$

(c) $5x + 4y + 3z - 6 = 0$

(d) $4x + 3y - 3z + 4 = 0$

(e) $5x - 3z - 8 = 0$

$$N = V \times W = \begin{vmatrix} i & j & k \\ 1 & 2 & -1 \\ 2 & 1 & 2 \end{vmatrix}$$

$$= (4 - (-1))i - (2 - (-2))j + (1 - 4)k$$

$$= \langle 5, -4, -3 \rangle$$

Using $P(1,1,-1)$ & N:

$$5(x-1) - 4(y-1) - 3(z+1) = 0$$

$$5x - 4y - 3z - 5 + 4 - 3 = 0$$

$$5x - 4y - 3z - 4 = 0$$

3. (15 points) Find the length of the curve $\mathbf{R}(t) = \langle 3t^2\,,\, 2t^2\,,\, -t^2\rangle$ over the interval $[0, 2]$.

(a) 12

(b) $8\sqrt{1\cancel{3}4}/3$

(c) $4\sqrt{1\cancel{3}4}$ *

(d) 16

(e) 32/3

$$R(t) = t^2\langle 3, 2, -1\rangle$$

$$R'(t) = 2t\,\langle 3, 2, -1\rangle$$

$$t \in [0,2]:\ \|R'(t)\| = 2t\sqrt{9+4+1}$$

$$= 2t\sqrt{14}$$

$$s = \int_0^2 |R'(t)|\,dt = \int_0^2 2t\sqrt{14}\,dt$$

$$= \sqrt{14}\,t^2\Big|_0^2$$

$$= 4\sqrt{14}$$

4. (15 points) Let $\mathbf{v}(t) = \langle t^2\,,\, e^{3t}\,,\, \cos t\rangle$ be the velocity vector and $\mathbf{R}(0) = \langle 1, -1, 2\rangle$ be the initial position vector. Find the position vector $\mathbf{R}(t)$.

(a) $\left\langle 2t+1, 3e^{3t}-1, -\sin t+2\right\rangle$

(b) $\left\langle \frac{t^3}{3}, \frac{e^{3t}-3}{3}, \sin t\right\rangle$

(c) $\left\langle t^2+1, e^{3t}-1, \cos t+2\right\rangle$

(d) $\left\langle \frac{t^3}{3}+1, \frac{e^{3t}-4}{3}, 2+\sin t\right\rangle$ *

(e) $\left\langle \frac{t^3}{3}+1, e^{3t}-2, 2-\sin t\right\rangle$

$$R(t) = R(0) + \int_0^t v(\tau)\,d\tau$$

$$= \left\langle 1 + \int_0^t \tau^2 d\tau, -1 + \int_0^t e^{3\tau} d\tau, 2 + \int_0^t \cos\tau\, d\tau\right\rangle$$

$$= \left\langle 1 + \frac{t^3}{3}, -1 + \frac{1}{3}(e^{3t}-1), 2 + \sin t\Big|_0^t\right\rangle$$

$$= \left\langle 1 + \frac{t^3}{3}, \frac{e^{3t}}{3} - \frac{4}{3}, 2 + \sin t\right\rangle$$

PART II. Work Out Problems. (Questions 5 – 6)

5. (20 points) Find the limit, if it exists

$$\lim_{x\to 0}\left\langle \frac{\sin(3x)}{2x}, \frac{3+x}{x^3-1}, \frac{x}{\ln(3x+1)} \right\rangle$$

$$= \left\langle \frac{3}{2}, \frac{3}{-1}, \frac{1}{3} \right\rangle$$

L'Hospital

$$\lim_{x\to 0} \frac{\sin 3x}{2x} = \lim_{x\to 0} \frac{3\cos 3x}{2} = \frac{3}{2}$$

$$\lim_{x\to 0} \frac{x}{\ln(3x+1)} = \lim_{x\to 0} \frac{1}{\frac{3}{3x+1}} = \frac{1}{3}$$

6. (20 points) Find the curvature $\kappa(t)$ of the following curve

$$R(t) = \left\langle \sin(3t) - 2,\, 2t,\, \cos(3t) + 2 \right\rangle$$

$$R' = \langle 3\cos 3t,\ 2,\ -3\sin 3t \rangle$$

$$R'' = \langle -9\sin 3t,\ 0,\ -9\cos 3t \rangle$$

$$R' \times R'' = \langle -18\cos 3t,\ -27,\ 18\sin 3t \rangle$$

$$= 9\langle -2\cos 3t,\ -3,\ 2\sin 3t \rangle$$

$$\|R'\| = \sqrt{9\cos^2 3t + 4 + 9\sin^2 3t} = \sqrt{9+4} = \sqrt{13}$$

$$\|R' \times R''\| = 9\sqrt{4\cos^2 3t + 4\sin^2 3t + 9} = 9\sqrt{13}$$

$$\kappa = \frac{\|R' \times R''\|}{\|R'\|^3} = \frac{9\sqrt{13}}{(\sqrt{13})^3} = \frac{9\sqrt{13}}{13\sqrt{13}} = \frac{9}{13}$$