Quiz 1

Fri 9/3/2010

$x_1 = 2,\ x_2 = 1\ \ x_3 = -1$

Solve the system

$$\begin{cases} x_1 - 2x_2 + x_3 = -1 \\ 2x_1 + x_2 - 3x_3 = 8 \\ 3x_1 - x_2 + 2x_3 = 3 \end{cases}$$

using the reduced echelon form.

Solution

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 2 & 1 & -3 & 8 \\ 3 & -1 & 2 & 3 \end{array}\right] \xrightarrow[(3)\to(3)-3(1)]{(2)\to(2)-2(1)} \left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 5 & -5 & 10 \\ 0 & 5 & -1 & 6 \end{array}\right]$$

$$\xrightarrow[(3)\to(3)-5(2)]{(2)\to(2)/5} \left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 4 & -4 \end{array}\right] \xrightarrow[\substack{(2)\to(2)+(3) \\ (1)\to(1)+2(2)-(3)}]{(3)\to(3)/4} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \overset{-1+2+1}{2} \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$$\begin{cases} x_1 = 2 \\ x_2 = 1 \\ x_3 = -1 \end{cases}$$

$(2, 1, -1)$.

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Friday 9/10/2010

Quiz 2

Calculate

12 a) $\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} - 3\begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & -7 \\ -4 & -2 \end{bmatrix}$

13 b) $\begin{bmatrix} 1 & 2 & -1 \\ 4 & -2 & 3 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 0 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 10 & -13 \end{bmatrix}$

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Quiz 3 Friday 9/17/2010

Find the inverse of the matrix

$\begin{bmatrix} 1 & 2 \\ 5 & 9 \end{bmatrix}$ by using Row operations.

Solutions Try $[A \mid I_2] \longrightarrow [I_2 \mid A^{-1}]$.

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 5 & 9 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{(2)\to(2)-5(1)} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -1 & -5 & 1 \end{array}\right]$$

$$\xrightarrow[(1)\to(1)-2(2)]{(2)\to -(2)} \left[\begin{array}{cc|cc} 1 & 0 & -9 & 2 \\ 0 & 1 & 5 & -1 \end{array}\right]$$

Hence the inverse is $\begin{bmatrix} -9 & 2 \\ 5 & -1 \end{bmatrix}$

Quiz 4. (Friday 10/1/2010)

Find the null space of matrix

$$A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 1 & 1 & 2 \\ 3 & 4 & -1 & -1 \end{bmatrix}.$$

<u>Note</u> this is subspace of $\mathbb{R}^4$

$$x = (x_1, x_2, x_3, x_4)$$

$$Ax = 0$$

$$\begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 1 & 1 & 2 \\ 3 & 4 & -1 & -1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & -2 & -1 & -10 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & -6 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 & -13 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & -6 \end{bmatrix}$$

$$x_1 = 13\alpha$$
$$x_2 = -8\alpha$$
$$x_3 = 6\alpha$$
$$x_4 = \alpha \text{ free}$$

$$\rightarrow N(A) = \left\{ \alpha \begin{pmatrix} 13 \\ -8 \\ 6 \\ 1 \end{pmatrix} \right\}.$$

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Quiz5 — Friday Oct 8, 2010.

Is the set $\{(1,2,3)^T, (1,1,-1)^T, (-2,-1,6)^T\}$ a Spanning set of $\mathbb{R}^3$.

<u>Solution</u> ~~Solve~~

$$\alpha_1 \underbrace{\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}}_{v_1} + \alpha_2 \underbrace{\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}}_{v_2} + \alpha_3 \underbrace{\begin{pmatrix} -2 \\ -1 \\ 6 \end{pmatrix}}_{v_3} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

for $\alpha_1, \alpha_2, \alpha_3$ with any a, b, c.

$$A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -1 \\ 3 & -1 & 6 \end{bmatrix} \qquad \alpha = (\alpha_1, \alpha_2, \alpha_3)^T$$

Then equation becomes

$$A\alpha = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

Augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & a \\ 2 & 1 & -1 & b \\ 3 & -1 & 6 & c \end{array}\right] \xrightarrow[(3)\to(3)-3(1)]{(2)\to(2)-2(1)} \left[\begin{array}{ccc|c} 1 & 1 & -2 & a \\ 0 & -1 & 3 & b-2a \\ 0 & -4 & 12 & c-3a \end{array}\right]$$

$$\xrightarrow[(3)\to(3)+4(2)]{(2)\to-(2)} \left[\begin{array}{ccc|c} \boxed{1} & 1 & -2 & a \\ 0 & \boxed{1} & -3 & 2a-b \\ 0 & 0 & 0 & c-3a+4(2a-b) \end{array}\right] \quad c-3a+4(2a-b) = 5a-4b+c$$

Take $a=b=c=1 \Rightarrow 5a-4b+c \neq 0$

$\Rightarrow$ No Solution $\alpha = (\alpha_1, \alpha_2, \alpha_3)^T$.

Hence $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \notin \text{Span}\{v_1, v_2, v_3\}$ and therefore $\{v_1, v_2, v_3\}$ is not a Spanning set of $\mathbb{R}^3$.

Quiz 6 Fri Oct 15

Let $f_1 = 2x$, $f_2 = \sin x$, $f_3 = \cos x$

Calculate the Wronskian $W[f_1, f_2, f_3](x)$ for $x \in \mathbb{R}$ and determine whether f_1, f_2, f_3 are linearly independent in $C^2(-\infty, \infty)$.

Solution

$$W(x) = \begin{vmatrix} 2x & \sin x & \cos x \\ 2 & \cos x & -\sin x \\ 0 & -\sin x & -\cos x \end{vmatrix}$$

$$= 2x(-\cos^2 x - \sin^2 x) - 2(-\sin x \cos x + \sin x \cos x)$$

$$= -2x.$$

$W(1) \neq 0 \rightarrow f_1, f_2, f_3$ are linearly INDEPENDEN