Solutions

MATH2360. SECTION 002. FALL 2009.

MIDTERM EXAMINATION 1

Name: .....

Signature: ..... Date: .....

READ AND FOLLOW THESE INSTRUCTIONS

This booklet contains 5 pages, including this cover page. Check to see if any are missing. PRINT all the requested information above, and sign your name. Put your initials on the top of every page, in case the pages become separated. Books and notes **are not permissible.** Calculators are allowed. Do your work in the blank spaces and back of pages of this booklet.

There are 4 work-out problems making a total score of 100 points. Students should show all the work in order to receive full credits. Unsupported answers will receive little credit.

AFTER YOU FINISH THE EXAM, turn in the whole booklet.

Problem 1 (25)	Problem 2 (15)	Problem 3 (30)	Problem 4 (30)	Total Score (100)

1. (25 points)

   (a) (15 points) Solve the following system of linear equations by using the augmented matrix and row operations to transform it into reduced row echelon form (Gauss-Jordan reduction):

$$x + 2y - z = 5$$
$$2x + 3z = -8$$
$$-x - y + 2z = -5$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 2 & 0 & 3 & -8 \\ -1 & -1 & 2 & -5 \end{array}\right] \xrightarrow[(3)\to(3)+(1)]{(2)\to(2)-2(1)} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 0 & -4 & 5 & -18 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{(2)\leftrightarrow(3)} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & -4 & 5 & -18 \end{array}\right] \xrightarrow{(3)\to(3)+4(2)} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 9 & -18 \end{array}\right]$$

$$\xrightarrow[\substack{(2)\to(2)-(3)\\(1)\to(1)-2(2)+(3)}]{(3)\to(3)/9} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5-2(2)+(-2)=-1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array}\right] \Rightarrow \begin{cases} x = -1 \\ y = 2 \\ z = -2 \end{cases}$$

   (b) (10 points) Solve the following system

$$x_1 + 2x_2 + 6x_4 = 1$$
$$x_2 + 3x_3 + 7x_4 = 0$$
$$x_3 + 2x_4 = -2$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 6 & 1 \\ 0 & 1 & 3 & 7 & 0 \\ 0 & 0 & 1 & 2 & -2 \end{array}\right] \xrightarrow[(1)\to(1)-2(2)]{(2)\to(2)-3(3)} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 4 & -1 \\ 0 & 1 & 0 & 1 & 6 \\ 0 & 0 & 1 & 2 & -2 \end{array}\right]$$

$$\begin{cases} x_1 = -1 - 4\alpha \\ x_2 = 6 - \alpha \\ x_3 = -2 - 2\alpha \\ x_4 = \alpha \text{ free} \end{cases}$$

2. (15 points) Let

$$A = \begin{pmatrix} 1 & 2 \\ -2 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 & 3 \\ 2 & -2 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 2 \\ -2 & 7 \\ 1 & 5 \end{pmatrix}$$

Calculate $A^2 - 2BC$.

$$A^2 = \begin{pmatrix} 1 & 2 \\ -2 & 5 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -2 & 5 \end{pmatrix} = \begin{pmatrix} 1-4 & 2+10 \\ -2-10 & -4+25 \end{pmatrix} = \begin{pmatrix} -3 & 12 \\ -12 & 21 \end{pmatrix}$$

$$BC = \begin{pmatrix} 1 & 0 & 3 \\ 2 & -2 & 1 \end{pmatrix}\begin{pmatrix} 0 & 2 \\ -2 & 7 \\ 1 & 5 \end{pmatrix} = \begin{pmatrix} 3 & 2+15 \\ 4+1 & +4-14+5 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 17 \\ 5 & -5 \end{pmatrix}$$

$$A^2 - 2BC = \begin{pmatrix} -3 & 12 \\ -12 & 21 \end{pmatrix} - 2\begin{pmatrix} 3 & 17 \\ 5 & -5 \end{pmatrix}$$

$$= \begin{pmatrix} -3-6 & 12-34 \\ -12-10 & 21+10 \end{pmatrix} = \begin{pmatrix} -9 & -22 \\ -22 & 31 \end{pmatrix}$$

3. (30 points)

Let

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & -2 \\ 0 & 5 & -8 \end{pmatrix}$$

(a) (25 points) Write the augmented matrix $[A|I_3]$ and use the row operations to transform it into $[I_3|A^{-1}]$ and find the inverse matrix A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & -1 & -2 & 0 & 1 & 0 \\ 0 & 5 & -8 & 0 & 0 & 1 \end{array}\right] \xrightarrow[(3)\to(3)+5(2)]{(1)\to(1)+2(2)} \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 & -1 & 0 \\ 0 & 0 & -18 & 0 & 5 & 1 \end{array}\right]$$

$(2)\to -(2)$

$$\xrightarrow[(2)\to(2)-2(3)]{(3)\to(3)/-18} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2-15/18 & 0-3/18 \\ 0 & 1 & 0 & 0 & -1-2(-\frac{5}{18}) & -2(-1/18) \\ 0 & 0 & 1 & 0 & -5/18 & -1/18 \end{array}\right]$$

$(1)\to(1)+3(3)$

$$\longrightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 21/18 & -3/18 \\ 0 & 1 & 0 & 0 & -8/18 & 2/18 \\ 0 & 0 & 1 & 0 & -5/18 & -1/18 \end{array}\right] \qquad A^{-1} = \begin{bmatrix} 1 & 7/6 & -1/6 \\ 0 & -4/9 & 1/9 \\ 0 & -5/18 & -1/18 \end{bmatrix}$$

(b) (5 points) Use the above inverse matrix to solve the equation

$$Ax = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$x = A^{-1}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{bmatrix} 1 & 7/6 & -1/6 \\ 0 & -4/9 & 1/9 \\ 0 & -5/18 & -1/18 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+7/6-1/6 \\ -4/9+1/9 \\ -5/8-1/8 \end{bmatrix} = \begin{bmatrix} 2 \\ -3/9 \\ -6/8 \end{bmatrix} = \begin{bmatrix} 2 \\ -1/3 \\ -3/4 \end{bmatrix}$$

4. (30 points)

Find determinants of the following matrices and determine whether they are singular or non-singular (invertible)

(a) (5 points)

$$\begin{pmatrix} 12 & 7 \\ -5 & 2 \end{pmatrix}$$

$\begin{vmatrix} 12 & 7 \\ -5 & 2 \end{vmatrix} = 12(2) - (-5)7 = 24 + 35 = 59$

non-singular

(b) (10 points)

$$\begin{pmatrix} 2 & 3 & -1 \\ 2+x & 3-2x & -1+4x \\ y & -2y & 4y \end{pmatrix}$$

where x and y are some given numbers.

$\begin{vmatrix} 2 & 3 & -1 \\ 2+x & 3-2x & -1+4x \\ y & -2y & 4y \end{vmatrix} = \begin{vmatrix} 2 & 3 & -1 \\ 2 & 3 & -1 \\ y & -2y & 4y \end{vmatrix} + \begin{vmatrix} 2 & 3 & -1 \\ x & -2x & 4x \\ y & -2y & 4y \end{vmatrix}$ Singular

$= 0 + xy \begin{vmatrix} 2 & 3 & -1 \\ 1 & -2 & 4 \\ 1 & -2 & 4 \end{vmatrix} = 0 + 0 = 0$

(c) (15 points)

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 5 & -4 \\ -1 & 0 & 1 \end{pmatrix}$$

use 3rd row

$\begin{vmatrix} 1 & 2 & 3 \\ -2 & 5 & -4 \\ -1 & 0 & 1 \end{vmatrix} = -1 \begin{vmatrix} 2 & 3 \\ 5 & -4 \end{vmatrix} - 0 + 1 \begin{vmatrix} 1 & 2 \\ -2 & 5 \end{vmatrix}$

$= -(-8-15) + 1(5+4)$

$= 23 + 9 = 32.$ non-singular