Tests About a Population Mean

MATH 3342

Section 8.2

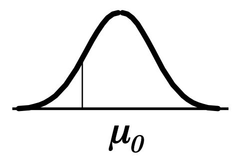
Formal Steps for a Hypothesis Test

1. State H_0 and H_a.
2. Calculate the **test statistic**.
3. Calculate the P-value.
4. Reach conclusion about H_0 using the decision rule.
5. State your conclusion in the context of your specific study.

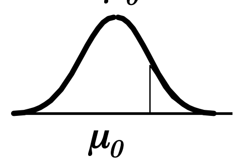
Three Sets of Hypotheses

Hypothesized value for the mean is μ_0.

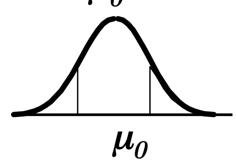
Lower Tail Test: H_0: $\mu = \mu_0$; H_a: $\mu < \mu_0$



Upper Tail Test: H_0: $\mu = \mu_0$; H_a: $\mu > \mu_0$



Two Tail Test: H_0: $\mu = \mu_0$; H_a: $\mu \neq \mu_0$



Formulating Hypotheses: Phone Example

- Suppose your company needs to reduce its mean cost, μ, for monthly phone usage.
- Currently, the mean is $3084 per month.
- Your company changes long-distance companies and institutes a new phone usage policy.
- To test if the changes succeeded:
  - **H_o: $\mu = 3084$**
  - **H_a: $\mu < 3084$**

Case I: A Normal Population with Known Variance

Assumptions for the Z-Test

- $X_1, X_2, \ldots, X_n$ are iid.
- The population is distributed Normally.
- The mean μ is unknown
- We **know** the population standard deviation σ.
  - Unlikely in practice.

The Z-Test

- The test statistic used is the Z-statistic

$$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

- The Z-statistic has a standard Normal distribution **when** H_o is true.

P-Values for the Z-test

Alternative Hypothesis	P-Value
$\mu > \mu_o$	$P(Z \geq z) = 1 - P(Z \leq z)$
$\mu < \mu_o$	$P(Z \leq z)$
$\mu \neq \mu_o$	$2*P(Z \geq \lvert z \rvert) = 2*P(Z \leq - \lvert z \rvert)$

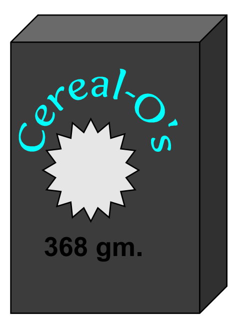
Example: Different Alternatives

If z = -2.1, find the p-value when:

a) H_a: $\mu > \mu_o$
b) H_a: $\mu < \mu_o$
c) H_a: $\mu \neq \mu_o$

Example: Cereal

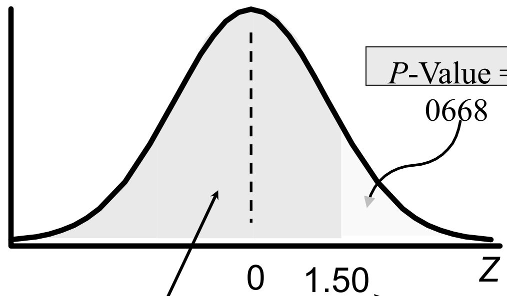
Q. Do boxes of cereal contain more than 368 grams of cereal on average? A random sample of 25 boxes showed $\bar{x}$ = 372.5. The company has specified σ to be 15 grams. Assuming Normality, test at the α = 0.05 level.



H_0: $\mu = 368$
H_a: $\mu > 368$

Example: Cereal

H_0: $\mu =$ 368
H_a: $\mu >$ 368



P-Value =.0668

1.0000
- .9332
.0668

From Table A: Lookup 1.50 to Obtain P(Z ≤ 1.50) = .9332

Observed Value of Z-statistic

P-Value =0.0668: Do Not Reject at α = .05

Example: Right-Handed

- In a simple random sample of 750 people, 658 people are right-handed.
- Use this data to test the claim that 90% of the population is right-handed with α=.10

Case II: Large-Sample Tests

Relaxed Assumptions

- For large n, the sample mean is approximately Normally distributed.
  - **The population does not need to be Normal.**
- We don't need to know σ, just s.
- Rule of thumb: $n > 40$

The Large-Sample Z-Test

- The test statistic used is the following:

$$Z = \frac{\bar{X} - \mu_0}{S / \sqrt{n}}$$

- Z has a approximately a standard Normal distribution **when** H_o is true.
- P-Values calculated as in Case I.
  - The significance level is now only ***approximately*** α.

Example: Quarters

- The U.S. Dept. of Treasury claims the mean weight of a quarter is 5.670 g.
- A SRS of 50 quarters has a mean weight of 5.652 g. and standard deviation of 0.068 g.
- Test the claim that the mean weight of quarters in circulation is 5.670 g at level α=0.10.

Case III: Small Samples from Normal Populations

Assumptions

- The population is Normally distributed.
- n is small.
  - **From rule of thumb, n < 40**
- σ is unknown.

Small-Sample Distribution

- Under these assumptions,

$$T = \frac{\bar{X} - \mu}{S / \sqrt{n}}$$

- Does NOT have a Normal distribution.
- It has a **t distribution** with *n – 1* ***degrees of freedom***.

Rejection Regions for the Z-Test

Alternative Hypothesis	Rejection Region
$\mu > \mu_o$	$t \geq t_{\alpha,n-1}$
$\mu < \mu_o$	$t \leq - t_{\alpha,n-1}$
$\mu \neq \mu_o$	Either $t \leq - t_{\alpha/2,n-1}$ Or $t \geq t_{\alpha/2,n-1}$

Example: Apartment Rent

A random sample of n = 10 apartment rents:

500,650,600,505,450,515,495,530,650,395

Is the mean rent greater than \$500? Let α= 0.05.

H_0: $\mu = 500$

H_a: $\mu > 500$

$\bar{x} = 531.0$

s = 82.8

n = 10

$$t = \frac{531.0 - 500}{82.8 / \sqrt{10}} = 1.184$$

Robustness

- An inference procedure is considered robust if the calculations required are **in**sensitive to violations in the assumptions of the procedure

Statistical Significance

- An event is said to be **statistically significant** if it is unlikely to occur by chance alone.
- If the P-Value < α, then our parameter of interest is significantly different than the value claimed in H_0.

Cautions

- There is no sharp border between "significant" and "insignificant," only increasingly strong evidence as the P-Value decreases.
- Statistical significance is not the same as practical significance.
- Badly designed experiments will produce useless results.
  - **It's important to know how the data was produced.**
- As always, be aware of outliers.