Hypotheses and Test Procedures

MATH 3342

Section 8.1

Is the claim wrong?

- An oil company representative claims that the average price for gasoline in Lubbock is \$3.20 per gallon.
- You think the average price is higher so you take a random sample and find that the mean is \$3.43 per gallon.
- Does this mean that the claim is wrong?

NOT NECESSARILY!

Hypothesis Testing

- A formal procedure in which we use sample data to test the plausibility of a hypothesis about:
  - The value of a parameter
  - The value of several parameters
  - An entire probability distribution
- We test a **null hypothesis** against an **alternative hypothesis**.

The Null Hypothesis

- The *null hypothesis* is denoted by H_0.
- Is a claim that is assumed to be true.
- Will be rejected only if the sample data provide substantial contradictory evidence.
  - Otherwise, continue to believe the plausibility of H_0
- The two possible conclusions of a test:
  - Reject H_0
  - Fail to reject H_0

The Alternative Hypothesis

- The *alternative hypothesis* is denoted as H_a.
- Is an assertion that is contradictory to H_0
- Often called the *researcher's hypothesis*
  - It is often a claim that a researcher would ultimately like to validate
- If we reject H_0, we do so in favor of H_a.

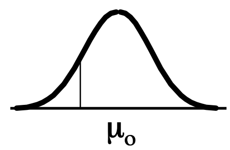
Analogy: Court Room

- One claim: defendant is not guilty
- Second claim: defendant ***is*** guilty
- In the US, we assume the defendant is not guilty and put the burden on the prosecution.

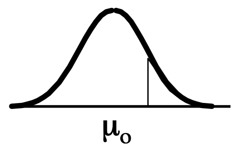
- Which is H_0?

Hypotheses for the Mean

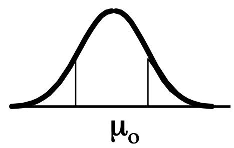
Lower Tail Test: $H_0: \mu = \mu_o$, $H_a: \mu < \mu_o$



Upper Tail Test: $H_0: \mu = \mu_o$, $H_a: \mu > \mu_o$



Two Tail Test: $H_0: \mu = \mu_o$, $H_a: \mu \neq \mu_o$



Example: Mice

- It takes mice a mean time of 18 seconds to find their way through a maze.
- A researcher thinks that a loud noise will cause them to complete the maze more quickly.
- She measures how long 10 mice take to complete the maze with noise as a stimulus.
  - H_o:
  - H_a:

Testing Procedure

1. State H_0 and H_a.
2. Calculate the **test statistic**.
3. Calculate the P-value.
4. Reach conclusion about H_0 using the decision rule.
5. State your conclusion in the context of your specific study.

The Test Statistic

- A function of the sample data.
- Provides a basis for testing a hypothesis.
- Measures compatibility between the data and the **null** hypothesis.
  - When H_o is true, we expect the data to closely agree with H_o.
  - When H_o is false, we expect the data to support H_a.

P-Values

- The probability of obtaining a test statistic at least as extreme as the test statistic we calculated from the sample.
  - Assuming H_o is true!!!
- The smaller the P-value, the stronger the evidence against H_o.

The Decision Rule

- If the P-Value is less than or equal to α, reject the **null** hypothesis in favor of the alternative.
  - $0 \leq \alpha \leq 1$
- If the P-value is greater than α, do **not** reject the null hypothesis.

How do we decide on a value for α?

- We want to limit the probability of making errors.
- Type I Error
  - Rejecting H_0 when it is true.
  - Probability denoted by α.
- Type II Error
  - Not rejecting H_0 when it is false.
  - Probability denoted by β.

Distinguishing the Type of Error

Truth About the Population

Decision Based on Sample

	H_o True	H_a True
Reject H_o	Type I Error	Correct Decision
Fail to Reject H_o	Correct Decision	Type II Error

Choosing the value of α

- Type I errors are typically more serious than Type II errors
- Approach is to select the largest value of α that can be tolerated.
- Choose a rejection region having that value of α rather than anything smaller.
- This value of α is called the **significance level** of the test.

Choosing the value of α

- The *significance level*:
  - The maximum probability of rejecting H_o when it is true.
- Common values of α:
  - 0.01, 0.05, 0.10