Confidence Intervals for the Variance of a Normal Population

MATH 3342
Section 7.4

The Distribution of S^2

- Let $X_1, X_2, \ldots, X_n$ be a random sample from a Normal population with parameters μ and σ^2.
- Then the RV

$$\frac{(n-1)S^2}{\sigma^2} = \frac{\sum (X_i - \bar{X})^2}{\sigma^2}$$

- Has a chi-squared (χ^2) distribution with $n - 1$ df

Critical Values

- Let $\chi^2_{\alpha,\nu}$ be the chi-squared **critical value**
- Denotes the number on the horizontal axis such that α of the area under the chi-squared curve with df ν lies to the right of $\chi^2_{\alpha,\nu}$
- Values are shown in Table A.7 in the Appendix

Derivation of a CI

$$P\left(\chi^2_{1-(\alpha/2),n-1} < \frac{(n-1)S^2}{\sigma^2} < \chi^2_{(\alpha/2),n-1}\right) = 1-\alpha$$

This is equivalent to:

$$P\left(\frac{(n-1)S^2}{\chi^2_{(\alpha/2),n-1}} < \sigma^2 < \frac{(n-1)S^2}{\chi^2_{1-(\alpha/2),n-1}}\right) = 1-\alpha$$

A 100(1- α)% CI for σ^2

- A 100(1 – α)% CI for σ^2 is as follows:

$$\frac{(n-1)s^2}{\chi^2_{(\alpha/2),n-1}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-(\alpha/2),n-1}}$$

- A 100(1 – α)% CI for σ is as follows:

$$\sqrt{\frac{(n-1)s^2}{\chi^2_{(\alpha/2),n-1}}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_{1-(\alpha/2),n-1}}}$$

Example

- The amount of lateral expansion was determined for n = 12 arc welds used in ship containment tanks.
- The resulting sample standard deviation was s = 3.56 mils.
- Assuming normality, derive a 95% CI for σ^2.