BASIC PRINCIPLES OF POINT ESTIMATION

MATH 3342
SECTION 6.1

POINT ESTIMATES

- A **point estimate** of a parameter θ :
  - A single number that can be regarded as a sensible value for θ
  - Obtained by selecting a suitable statistic and computing its value from the given sample data.
- The selected statistic is called the **point estimator** of θ
- Both the estimate and estimator are denoted by $\hat{\theta}$

HOUSING PRICE EXAMPLE

All Values Given in Thousands of Dollars

{House Prices} = {144; 98; 204; 177; 155; 316; 100}

$$\bar{x} = 170.6$$

$$\tilde{x} = 155$$

$$\bar{x}_{14.3\%} = 156$$

All of these are point estimates of μ !!!

How do we know which one to use?

UNBIASED ESTIMATORS

- A point estimate $\hat{\theta}$ is said to be an **unbiased estimator** of θ if:

$$E(\hat{\theta}) = \theta \text{ for all possible values of } \theta$$

- If $\hat{\theta}$ is not unbiased, the difference:

$$E(\hat{\theta}) - \theta \text{ is called the bias of } \hat{\theta}$$

THE PRINCIPLE OF UNBIASED ESTIMATION

- When choosing among several different estimators of θ, choose one that is unbiased.
- Example:
- This is why we use $\hat{\sigma}^2 = S^2 = \dfrac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$,
- Not $\dfrac{n-1}{n}S^2 = \dfrac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n}$

RESULT

- If $X_1,\ldots,X_n$ is a random sample from a distribution with mean μ, then the sample mean is an unbiased estimator of μ.
- If, additionally, the distribution is *continuous* **and** *symmetric*, then the sample median and any trimmed mean are also unbiased estimators of μ.

ESTIMATORS WITH MINIMUM VARIANCE

- Among all estimators of θ that are unbiased, choose the one that has minimum variance.
- The resulting $\hat{\theta}$ is called the **minimum variance unbiased estimator (MVUE)** of θ
- See pages 246 – 248 for an example.

THEOREM

- Let $X_1,\ldots,X_n$ be a random sample from a normal distribution with mean μ and standard deviation σ.
- Then:

$$\hat{\mu} = \bar{X} \text{ is the MVUE for } \mu$$

STANDARD ERROR

- The **standard error** (SE) for an estimator is its standard deviation.

$$\sigma_{\hat{\theta}} = \sqrt{V(\hat{\theta})}$$

- The usual measure of precision for an estimator.
- Often, SE depends on unknown parameters, so we must use **estimated standard error** instead.
  - Denoted by $\hat{\sigma}_{\hat{\theta}}$ or $s_{\hat{\theta}}$

COMPLICATIONS

- Choice of point estimate can depend on the population distribution.
  - See page 249 for an example.
- Sometimes, standard error can not be calculated using standard techniques, so a type of simulation called the **bootstrap** must be used instead.
  - As is the case with calculating the SE for S^2.
  - The basic idea is described on pages 251 – 252.