The Gamma Family of Distributions

MATH 3342
Section 4.4

The Gamma Family

- The Gamma Distribution
- The Exponential Distribution
- The Chi-Squared Distribution

- Skewed distributions
- Do not all have the same shape

The Gamma Distribution

- A continuous RV X has a **gamma distribution** if it has the following pdf:

$$f(x;\alpha,\beta) = \left\{ \begin{array}{ll} \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{x/\beta}, & x \geq 0 \\ 0 & otherwise \end{array} \right\}$$

- Where $\Gamma(\alpha)$ is as defined on the following slide.

$$\alpha > 0, \beta > 0$$

$$E(X) = \alpha\beta \qquad V(X) = \alpha\beta^2$$

The Gamma Function

- For α > 0, the **gamma function** is defined as:

$$\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x}\, dx$$

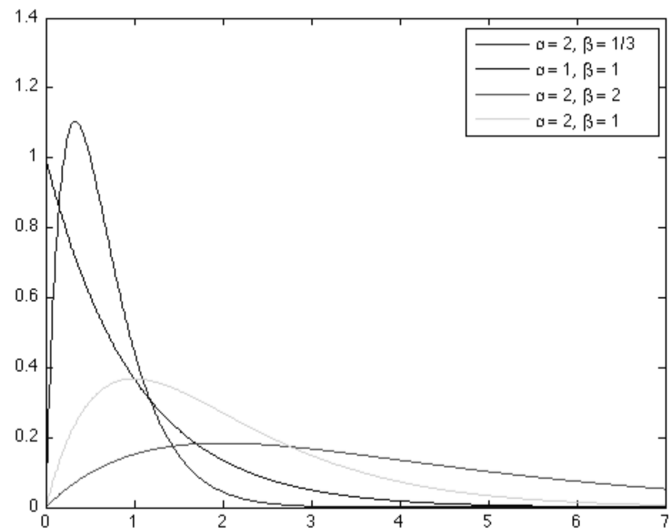
- Important properties:

For $\alpha > 0, \Gamma(\alpha) = (\alpha - 1) \cdot \Gamma(\alpha - 1)$

For any positive integer $n, \Gamma(n) = (n-1)!$

$\Gamma(\frac{1}{2}) = \sqrt{\pi}$

Gamma Density Curves



The Exponential Distribution

- A continuous RV X has an **exponential distribution** if it has the following pdf:

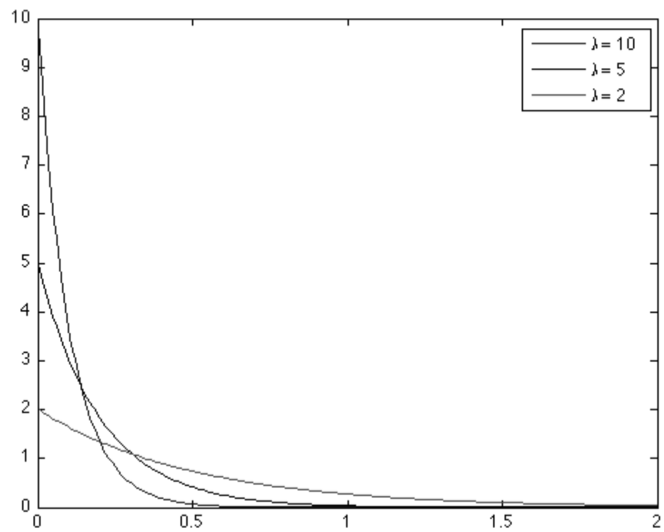
$$f(x;\lambda) = \begin{Bmatrix} \lambda e^{-\lambda x}, & x \geq 0 \\ 0 & otherwise \end{Bmatrix}$$

- The cdf is:

$$F(x;\lambda) = \begin{Bmatrix} 0 & x < 0 \\ 1 - e^{-\lambda x}, & x \geq 0 \end{Bmatrix}$$

$$E(X) = \frac{1}{\lambda} \qquad V(X) = \frac{1}{\lambda^2}$$

Exponential Density Curves



The Chi-Squared Distribution

- A continuous RV X has a **chi-squared** (χ^2) **distribution** if it has the following pdf:

$$f(x;\alpha,\beta) = \begin{cases} \dfrac{1}{2^{\nu/2}\Gamma(\nu/2)} x^{\nu/2-1} e^{x/2}, & x \geq 0 \\ 0 & otherwise \end{cases}$$

- It is a Gamma distribution with α = ν/2 and β=2.
- The parameter ν is called the degrees of freedom of X

$$E(X) = \nu \qquad V(X) = 2\nu$$

- Important distribution for many types of statistical inference.