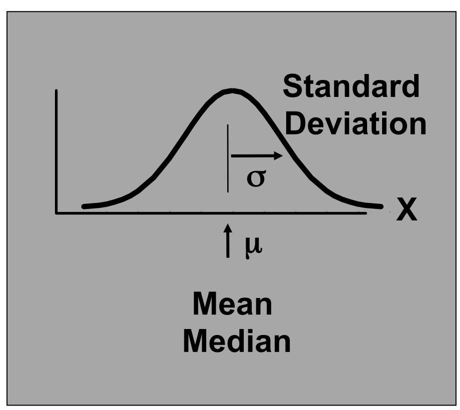
The Normal Distribution

MATH 3342
Section 4.3
(& a Small Part of Section 4.6)

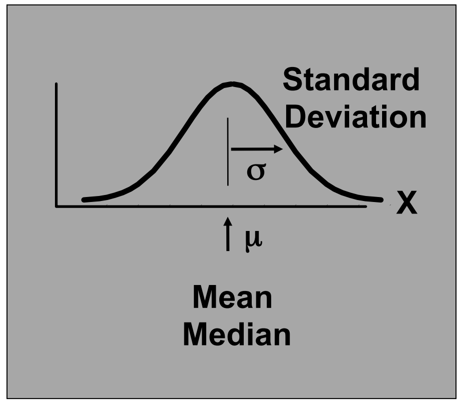
Normal Distributions

- "Bell shaped"
- Symmetric
- Mean and median of the curve are equal

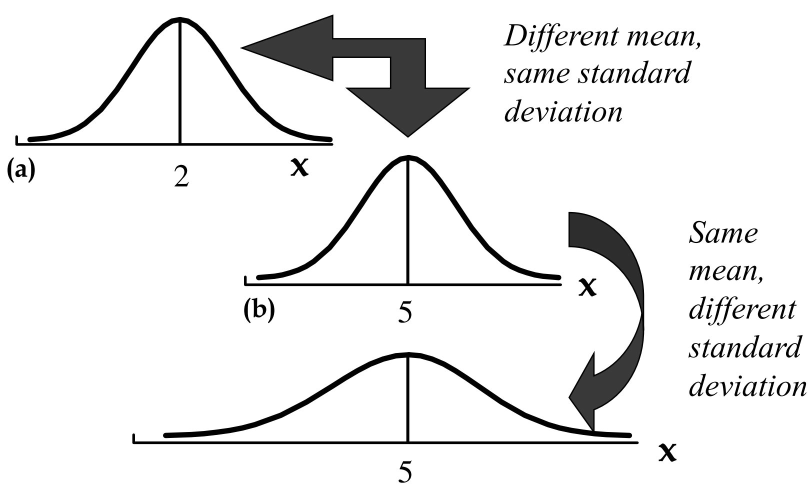


Normal Distributions

- Data values range from -∞ to +∞
- Completely described using only μ and σ
- Denoted by: $N(\mu,\sigma^2)$



Differences Between Normal Distributions



Pdf for the Normal Distributions

- A continuous RV X has a Normal distribution with parameters - ∞ < μ < ∞ and σ > 0 if it had the following pdf:

$$f(x;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/(2\sigma^2)},$$

$$-\infty < x < \infty$$

- There is no closed form solution for the cdf.

The Standard Normal Distribution

- The *standard normal distribution* is a normal distribution where
  - μ= 0
  - σ= 1
- Measures the number of standard deviations a point is from the mean.
- Positive z-values are above the mean and negative z-values are below.

pdf for the Standard Normal Distribution

- A RV is said to be a **standard Normal RV** and is denoted by Z if it has the following pdf:

$$f(z;0,1) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

- The cdf is denoted by $\Phi(z)$

Standard Normal Percentiles

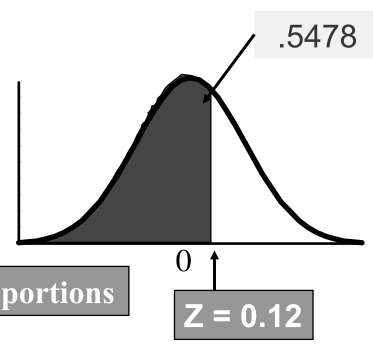
- There is no formula for calculating these.
- To obtain them, either software or ***tables*** are used.
- Table A provides the percentiles for Z.
  - Referred to as Table A.3 in the book.

Using Table A

Partial Cumulative Standardized Normal Distribution Table

Z	.00	.01	**.02**
0.0	.5000	.5040	.5080
0.1	.5398	.5438	.5478
0.2	.5793	.5832	.5871
0.3	.6179	.6217	.6255

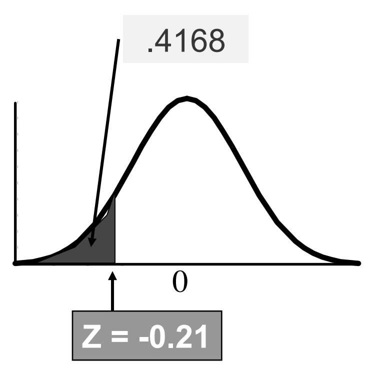
Proportions



Using Table A

Part of Table A

Z	.00	.01	**.02**
-.30	.3821	.3783	.3745
-.20	.4207	.4168	.4129
-.10	.4602	.4562	.4522
0.0	.5000	.4960	.4920



Example

Find the proportions corresponding to the following statements:

a) $z < 2.85$

b) $z > 2.85$

c) $z > -1.66$

d) $-1.66 < z < 2.85$

Standardizing

- If a variable *x* is from $N(\mu,\sigma^2)$, then the standardized value of x, called a **z-score** is the following:

$$z = \frac{x - \mu}{\sigma}$$

- The variable *z* is from N(0,1)

Example

Weights of baby elephants X follow a Normal distribution with mean μ=224 lbs and standard deviation σ=53 lbs.

a) $P(X < 200 \text{ lbs}) = ?$

b) $P(200 < X < 300 \text{ lbs}) = ?$

c) $P(X > 445) = ?$

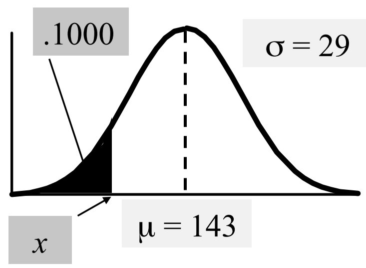
Finding a Percentile for any Normal Distribution

1. State the problem and draw a picture.
   What are the mean and standard deviation? What percentile is desired?
2. Use Table A.
   Look for the entry closest to the given probability to find the z-score.
3. Unstandardize the z-score.
   Transform *z* back to the desired *x* scale.

Example

The steel reinforcement bars manufactured in a foundry have lengths that follow a normal distribution with mean μ =143 in. and standard deviation σ = 29 in.

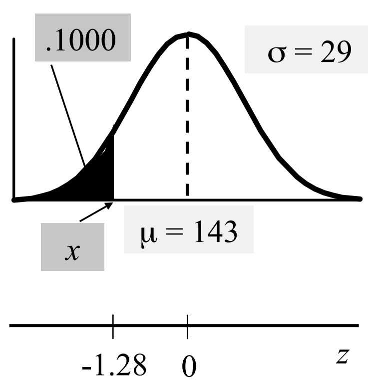
Below what length would 10% of the bars fall?



Example

- **Use Table A to find the z score corresponding to the region bounded by *x*.**

Z	.07	**.08**	0.9
-1.5	.0582	.0571	.0559
-1.4	.0708	.0694	.0681
-1.3	.0853	.0838	.0823
-1.2	.1020	**.1003**	.0985



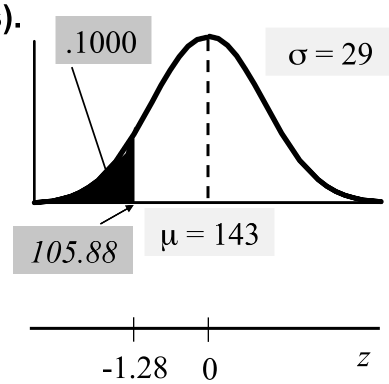
Example

- Use the following formula to translate the *z* score into *x* (in the original units).

$$x = \mu + z\sigma$$

$$x = 143 + (-1.28)\,29$$

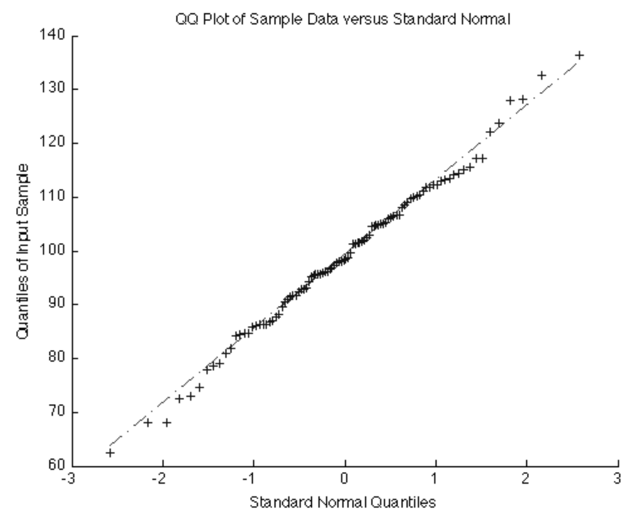
$$x = 105.88 \text{ in.}$$



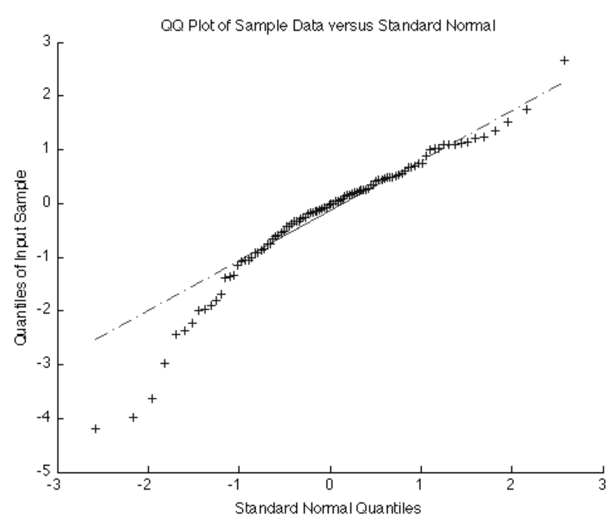
Normal Probability Plots

- Plots sample percentiles against percentiles of the standard Normal distribution
- Provides method for determining whether an assumption of Normality is plausible:
- If the sample comes from $N(\mu,\sigma^2)$, the points should fall close to a line with slope σ and intercept μ.

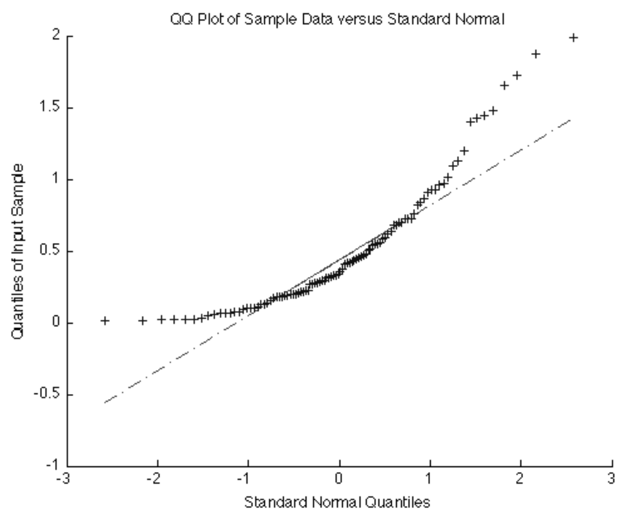
Example



Example



Example


QQ Plot of Sample Data versus Standard Normal
Quantiles of Input Sample
Standard Normal Quantiles
2
1.5
1
0.5
0
-0.5
-1
-3
-2
-1
0
1
2
3