ADDITIONAL DISCRETE DISTRIBUTIONS

Math 3342
Sections 3.5 and 3.6

Related to the Binomial Distribution

- Hypergeometric
  - Probability of X successes, samples with***out*** replacement
  - Binomial samples ***with*** replacement (independent trials)
- Negative Binomial
  - Counts number of trials needed to observe some number of successes
  - Binomial has fixed number of trials

Hypergeometric Setting

- The **population** consists of *N* individuals/objects
  - Finite population!
- Each individual is either a success S or failure F
- There are M successes in the population
- A sample of *n* individuals is selected with***out*** replacement
  - Done such that each sample of size *n* is **equally likely** to be chosen

The Hypergeometric pmf

- Let X = # of successes in a random sample of size *n*
- Sample is drawn from a population consisting of:
  - *M* successes
  - *N* – *M* failures
- The pmf for X is given by:

$$P(X = x) = h(x; n, M, N) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$$

- For max(0,*n*-*N*+*M*) ≤ *x* ≤ min(*n*,*M*)

Hypergeometric Mean and Variance

$$E(X) = n \cdot \frac{M}{N}$$

$$V(X) = \left(\frac{N-n}{N-1}\right) \cdot n \cdot \frac{M}{N} \cdot \left(1 - \frac{M}{N}\right)$$

Example: Elephants

- 6 elephants were caught and tagged, then released to the wild.
- After giving an opportunity to remix, a sample of 12 elephants is selected.
- Let X = # of tagged animals in the second sample.
- If the number of elephants in this population is 30:
- What is the probability that X = 3?
- What are E(X) and V(X)?

Negative Binomial Setting

- The random process consists of a sequence of *independent* trials.
- Each individual is either a success S or failure F.
- The probability of success is constant from trial to trial.
- Trials are performed until a total of *r* successes have been observed.

The Negative Binomial Distribution

- X is a RV denoting the # of failures that precede the *r*th success.
- Possible values of X are 0, 1, 2,…
- The pmf for this distributed is as follows:

$$P(X = x) = nb(x; r, p) = \binom{x + r - 1}{r - 1} p^r (1 - p)^x$$

$$E(X) = \frac{r(1-p)}{p} \quad V(X) = \frac{r(1-p)}{p^2}$$

Example: Clinical Trial

- A research hospital wishes to recruit 7 patients suffering from a chronic disease for a clinical trial for a new treatment.
- Let p = 0.6 be the probability that a patient agrees to participate.
- What is the probability that 9 patients must be asked?
  - That is, what is P(X = 2)?
- What are E(X) and V(X)?

Example: Fish Trapping

- Let X denote the number of fish that a trapping device captured within a given time period.
- On average, this trap typically catches 6 fish in that time period.
- What is the probability that at the end of the time period, the trap contains exactly 2 fish?
  - 2 or fewer fish?

The Poisson Distribution

- A discrete RV X which has a Poisson distribution with parameter μ is described by the following pmf:

$$P(X = x) = p(x;\mu) = \frac{e^{-\mu}\mu^{x}}{x!}$$

for x = 0, 1, 2, 3, … .

$$E(X) = V(X) = \mu$$

Example

- Let X denote the number of fish that a trapping device captured within a given time period.
- On average, this trap typically catches 4 fish in that time period.

- What is the probability that at the end of the time period, the trap contains exactly 2 fish?
  - 2 or fewer fish?

The Poisson Process

- Important application of the Poisson distribution
- Used in connection with the occurrence of events over time.
- Examples:
  - Visits to a particular website
  - Electromagnetic pulses recorded by a counter
  - Accidents in an industrial factory