Tests About a Population Mean

MATH 3342

Sections 9.4 and 9.5, Part 1

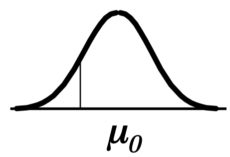
Formal Steps for a Hypothesis Test

1. State H_0 and H_a.
2. Calculate the **test statistic**.
3. Determine the **critical value** to define the **rejection region**.
4. Reach conclusion about H_0.
5. State your conclusion in the context of your specific study.

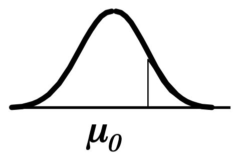
Three Sets of Hypotheses

Hypothesized value for the mean is μ_0.

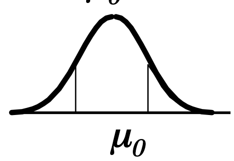
Lower Tail Test: $H_0: \mu = \mu_0$, $H_a: \mu < \mu_0$



Upper Tail Test: $H_0: \mu = \mu_0$, $H_a: \mu > \mu_0$



Two Tail Test: $H_0: \mu = \mu_0$, $H_a: \mu \neq \mu_0$



Formulating Hypotheses: Phone Example

- Suppose your company needs to reduce its mean cost, μ, for monthly phone usage.
- Currently, the mean is $3084 per month.
- Your company changes long-distance companies and institutes a new phone usage policy.
- To test if the changes succeeded:
  - **H_o: $\mu = 3084$**
  - **H_a: $\mu < 3084$**

Case I: A Normal Population with Known Variance

Assumptions for the Z-Test

- The observations are from a SRS.
- The population is distributed Normally.
- The mean μ is unknown
- Either:
  - We **know** the population standard deviation σ.
    - Unlikely in practice.
  - OR the sample size is large ($n > 40$)

The Z-Test

- The test statistic used is the Z-statistic

$$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

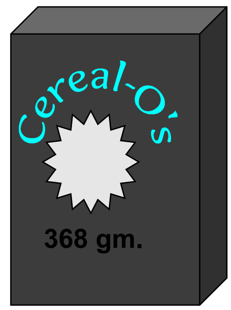
- The Z-statistic has a standard Normal distribution **when** H_o is true.

Rejection Regions for the Z-Test

Alternative Hypothesis	Rejection Region
$\mu > \mu_o$	$z \geq z_{\alpha}$
$\mu < \mu_o$	$z \leq - z_{\alpha}$
$\mu \neq \mu_o$	Either $z \leq - z_{\alpha/2}$ or $z \geq z_{\alpha/2}$

Example: Cereal

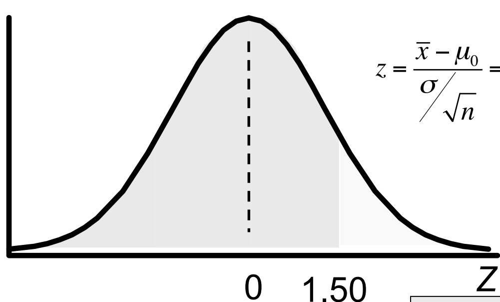
Q. Do boxes of cereal contain more than 368 grams of cereal on average? A random sample of 25 boxes showed $\overline{x} = 372.5$. The company has specified σ to be 15 grams. Assuming Normality, test at the $\alpha = 0.05$ level.



H_0: $\mu = 368$
H_a: $\mu > 368$

Example: Cereal

H_0: $\mu = 368$
H_a: $\mu > 368$



$$z = \frac{\overline{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{372.5 - 368}{15 / \sqrt{25}} = \frac{4.5}{3} = 1.5$$

Value of Test Statistic

$z = 1.5 < 1.645 = z_{.05}$

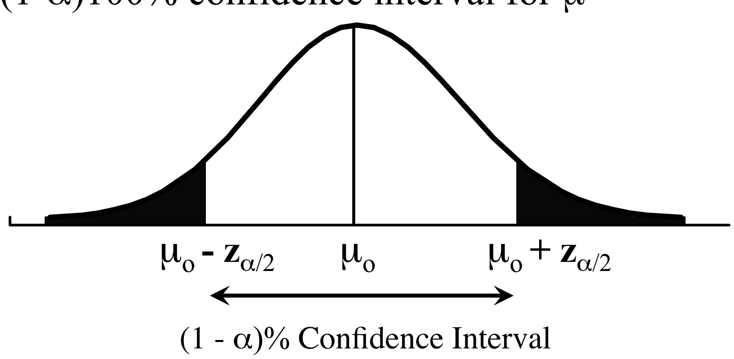
Do Not Reject at α = .05

Example: Quarters

- The U.S. Dept. of Treasury claims the mean weight of a quarter is 5.670 g.
- A SRS of 50 quarters has a mean weight of 5.652 g. and standard deviation of 0.068 g.
- Test the claim that the mean weight of quarters in circulation is 5.670 g at level α=0.10.

Two-Tailed Tests and Confidence Intervals

- A level α two-sided significance test **rejects** H_o exactly when the value μ_o falls outside a level $(1-\alpha)100\%$ confidence interval for μ



Case III: Small Samples from Normal Populations

Assumptions

- The population is Normally distributed.
- *n* is small.
  - **From rule of thumb, *n* < 40**
- σ is unknown.

Small-Sample Distribution

- Under these assumptions,

$$T = \frac{\bar{X} - \mu}{S / \sqrt{n}}$$

- Does NOT have a Normal distribution.
- It has a **t distribution** with *n – 1**degrees of freedom***.

Rejection Regions for the Z-Test

Alternative Hypothesis	Rejection Region
$\mu > \mu_o$	$t \geq t_{\alpha,n-1}$
$\mu < \mu_o$	$t \leq - t_{\alpha,n-1}$
$\mu \neq \mu_o$	Either $t \leq - t_{\alpha/2,n-1}$ Or $t \geq t_{\alpha/2,n-1}$

Example: Apartment Rent

A random sample of n = 10 apartment rents:

500,650,600,505,450,515,495,530,650,395

Is the mean rent greater than \$500?
Let α= 0.05.

H_0: $\mu = 500$

H_a: $\mu > 500$

$\bar{x} = 529.0$

s = 82.5

n = 10

$$t = \frac{529.0 - 500}{82.5/\sqrt{10}} = 1.112$$

Other Options

- Possible that none of these cases are appropriate.
- Could use a distribution-free test.
  - **Also called nonparametric tests**
- Could work with a statistician to use procedures developed for specific families of distributions.
- Could develop a procedure based on computer simulations.